



Proceedings of the 41st Conference of the International Group for the Psychology of Mathematics Education

Editors

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Volume 4

PME 41, Singapore, 17-22 July 2017



Cite as:

Kaur, B., Ho, W.K., Toh, T.L., & Choy, B.H. (Eds.). (2017). Proceedings of the 41st Conference of the International Group for the Psychology of Mathematics Education (Vol. 4). Singapore: PME.

Website: <http://math.nie.edu.sg/pme41>

Proceedings are also available via <http://www.igpme.org>

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ISBN: 978-981-11-3742-6

Cover design: Mohamed Fadzli Ibrahim, National Institute of Education, Singapore.

Printed by Welon Printing Co. Pte Ltd

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EXAMINING MATHEMATICAL SOPHISTICATIONS IN COLLABORATIVE PROBLEM SOLVING

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This paper reports on efforts to characterise levels of mathematical sophistication for students in collaborative mathematics problem solving. Using a laboratory classroom in Australia, data were captured with multiple cameras and audio inputs. Students worked individually, in pairs, and in small groups (4 to 6 students). We focused on investigating collaborative work, with the goal of studying the mathematical sophistications of students' reasoning when solving problems. Drawing from two analytical frameworks to document the mathematical sophistication in students' exchange, levels of cognitive demands and mathematical practices, this research highlights different aspects of students' reasoning in solving these tasks.

SOCIAL ENVIRONMENT FOR LEARNING

In social settings, learning involves complex processes including teacher-student and student-student interactions. Research designs in such settings need to be sensitive to the multifaceted nature of learning (Clarke et al., 2012). During collaborative problem solving of open-ended tasks, students have to negotiate approaches to a task as a group, which obliges the students to articulate their thinking overtly and this can make visible the learning processes. This study is part of a bigger project that investigates social interactions in learning through a research design that focuses on collaborative problem solving in mathematics. Available research facilities in Australia capture different sources of data including videos and audios records, as well as student artefacts. This paper specifically focuses on applying two approaches for documenting mathematical sophistication in students' reasoning in the classroom setting. The analysis reported in this paper addresses the research question: What are the levels of mathematical sophistication (in written product and in spoken interaction) displayed by individuals and groups in the social unit (pair and small group) as they solve open-ended mathematical tasks?

RELATED LITERATURE

Given the focus of this paper is on students working collaboratively on real-world mathematical problems, we have examined related works drawing on research on problem solving prior to 1990 including problem difficulties and characteristics of problem solvers (cf. Lesh & Zawojewski, 2007). One line of research exclusively focuses on features of tasks for students to solve in school. According to Lester and Kehle (2003), these task features include content and context, structure, syntax, and heuristic behaviour variables. Lesh and Zawojewski commented that still missing in

this line of research is the consideration of the interactions between task difficulties and the characteristics of the problem solver. In other words, how students respond to tasks as a result of their personal characteristics matters. When solving problems, tasks alone do not account for how problem solvers interpret the same task differently. Students' interpretation of tasks depends not only on task characteristics (e.g., mathematical content, figurative task context, levels of cognitive demand), but also on characteristics of the learner and the class (i.e., cognition and affect) (Lesh & Zawojewski, 2007). A second line of research distinguishes between good and poor problem solvers. Lester and Kehle (2003) summarized that, (a) good problem solvers know more than the poor ones and their knowledge is well organised, not in discrete form but as a structured and connected network, and (b) the attention of good problem solvers is on the structure of the problems, while poor problem solvers focus on irrelevant information and the surface features of the problems. In this study, we did not aim to document task difficulties or to distinguish types of problem solvers in term of the novice-expert paradigm. Instead, we focused on what students do in the setting as they solve the problems and we documented evidence of their mathematical sophistication. Arguably, students' responses are dependent on the task variables, therefore the focus of this paper lies in the interaction between the two lines of research identified above.

Researchers such as Stein and Lane (1996) have emphasized the role of instructional tasks as catalysts for student learning. Stein and Lane conceptualise tasks as passing through three phases: (a) as represented in curriculum/instructional materials, (b) as set up by the teacher in the classroom, and (c) as implemented by students during the lesson. This study focused on what students do when they are working on problem solving tasks, therefore it could be considered as addressing the third phase of task implementation. Furthermore, tasks can be examined for their cognitive demand—the kinds of thinking processes that are required in solving each task. Stein and Lane found that the cognitive demand required by tasks influence student learning because they determine the ways students think about, develop, and use mathematics. Their framework presents four levels of cognitive demand: *memorization*, *procedure without connection*, *procedure with connection*, and *doing mathematics*. This framework was adopted in this study to focus on what students do when facing such tasks. From the perspective of the cognitive demand of tasks we could deduce what was required of students by each task and compare this to what the students actually did when attempting the tasks. For example, in the high level of *doing mathematics*, tasks require complex and non-algorithmic thinking to provide the opportunity for students to execute such thinking in the setting.

An alternative way to look at mathematical sophistication is through documenting how different mathematical practices (CCSSI, 2010) or mathematical habits of mind (Cuoco, Goldenberg, & Mark, 1996) are performed when students solve mathematical problems. Building on mathematical proficiencies (Kilpatrick, Swarfford, & Findell, 2001) and National Council of Teachers of Mathematics (NCTM) process standards

(2000), eight standards of mathematical practices were formulated representing the process that mathematicians and students carry out when they are doing mathematics. These practices include: (a) Make sense of problems and persevere in solving them. (b) Reason abstractly and quantitatively. (c) Construct viable arguments and critique the reasoning of others. (d) Model with mathematics. (e) Use appropriate tools strategically. (f) Attend to precision. (g) Look for and make use of structure (h) Look for and express regularity in repeated reasoning. Further details for the practices could be found in CCSSI (2010). In this study, these eight mathematical practices were used to investigate what features of the practices are evident when the students were interacting with each other when attempting the problem solving tasks. Together, the two analytic frameworks provide complementary perspectives to capture the nuances of mathematical sophistications in students' reasoning.

METHODOLOGY

The Setting

The recent development of a laboratory classroom, the Science of Learning Research Classroom (SLRC) at the University of Melbourne has made possible research designs that provide a better approximation to natural social settings, while allowing researchers to retain some control over aspects of the setting. In the Social Unit of Learning project, which utilised the SLRC for data collection, students work individually, in pairs, or in groups with their usual teacher. Yet, researchers could control task characteristics, the level of intervention from teachers, and possible forms of social interactions. With 10 built-in video cameras and up to 32 audio channels, the SLRC has the capability to capture classroom social interactions with a rich amount of detail. The facility was purposefully designed to allow simultaneous and continuous documentation of classroom interactions. The Social Unit of Learning project collected multiple forms of data including student written products and high definition video and audio recordings of every student and the teacher in the classroom. Intact Year 7 classes were recruited with their usual teacher for the project in order to exploit existing student-student and teacher-student interactive norms. Each class participated in a 60-minute session in the laboratory classroom involving separate problem solving tasks that required them to produce written solutions.

Problem Solving Tasks

To make the meaning negotiation process of the students visible for observation, open-ended tasks were chosen to allow students to have multiple entry points and require students to interact. Such tasks also call for different representations including numerical, symbolic, and graphical. In addition, the tasks afforded connection to contexts outside the classroom in order to facilitate discussion. These tasks were drawn from previous research (e.g., Sullivan & Clarke, 1992) and have been found to create opportunities for students to reason and to articulate their thinking. In the session analysed in this study, the three tasks included content foci that were disconnected to avoid carry-over effects between tasks. Task 1 focused on students' abilities to make

sense of information from an incomplete graphical display – a bar graph. Students need to interpret what it is about and create the story from the graph. In Task 2, students were given an average age of the people in a household for which one person's age is constrained but requires interpretation and were asked to figure out the age of the other people in the household as well as the relationship between them. For the last task, students were required to work out the plan for a five-room apartment, which has a total area of 60 square metres. The students attempted the first task individually (10 minutes), the second task in pairs (15 minutes), and the third task in groups of four to six students (20 minutes).

The wording Task 2, used in this study is as follows:

Task 2: "The average age of five people living in a house is 25. One of the five people is a Year 7 student. What are the ages of the other four people and how are the five people in the house related? Write a paragraph explaining your answer."

Data Analysis

Two frameworks were used for coding data: one related to levels of cognitive demands, and another to mathematical practices. The cognitive demand framework (Stein & Lane, 1996) was adapted to describe what students do when facing the cognitive demands of tasks. Next, specific observations related to eight mathematical practices (CCSSI, 2010) were undertaken to help guide the coding of transcript and student artefact data. Videos of students working on the three problem solving tasks and the associated transcripts were used as a primary source for data analysis. After watching the videos and reading the transcripts, we created a mathematical story line to document their problem solving process. The students' written work was referenced occasionally to help explain their talk in the transcript. After creating the story lines, we then mapped the students' actions onto the two frameworks: levels of cognitive demand and mathematical practices. For levels of cognitive demand, which are hierarchical in nature, we observed what was going on in the discussion and how mathematical reasoning was developed during problem solving. A level was considered to have been attained if the student(s) illustrated at least one of the criteria appropriate to that level. Furthermore, when several levels were observed, the highest level was coded. For the coding of mathematical practices, each of the mathematical practices was documented when performed together with the time that the practice occurred.

PRELIMINARY FINDINGS

Initial observation suggests that when students work individually, barely any conversation happened and students rarely talked aloud. The main source of data for Task 1 (individual work) was their written work. Therefore, in this paper, we will illustrate how the coding was employed for a pair of students when they worked on Task 2 (pair work). This paper illustrates its key points by drawing on the written solutions, transcripts, and video record from one pair, two male students. John and Arman, working on Task 2. First, a mathematical story line, a narrative of student's

mathematical reasoning when solving the task, was constructed by one of the researchers for John and Arman. John, an English language learner, had some difficulty understanding mathematical and non-mathematical words in the written task. This seemed to restrict his entry into the task. Notwithstanding, he asked about the meaning of the words *related* and *average* and strived to make sense of the task. The teacher explained to him the meaning of *related*, but not the meaning of *average*. When John approached his peer Arman, Arman provided different unclear descriptions of *average*.

Arman: You know what average is? Average is - average age of five people living in a house is... It's like the maximum.

John: Huh?

Arman: Maximum age.

John: Oh. What's it mean? Okay, okay. Is not - important but ...

Arman: Okay. Average is like the most likely so most of the people in the - so most of five people living in a house is 25.

John was able to identify the age of the Year 7 student in the house as 12 years old, and tasked himself to find the ages of other four people. As he still had problems with understanding the concept of average, he also had difficulty elaborating what he was looking for: "Is five people the - which is together is 25 or each person is 25?" (John). He persisted with solving the task as he got more information from his peer.

John: Yeah. So you just guess the person of - no, it's a - how to say it? Just bigger than 13.

Arman: Yeah. So 12 and 25, lower - lower - younger than 25 and older than 12.

John: Are they same age or different?

Arman: Okay. So one can be 17 ...

John: Yeah.

Arman: ... yeah? Another one can be 14, 15.

John: I don't think so.

Arman: Or it can be older.

John: Oh... (moaning and groaning helplessly) Just - just don't like to [inaudible] (laughs).

Arman re-read the problem and picked out critical information from the task: what was the given information (one person was in Year 7), the household average age of 25 years and that the Year 7 student was not 25 years old, "Year 7 student is not 25 years, right?"), and what was being asked (age of each of the five people and their relationship). Arman interpreted average as maximum and typical, "most likely" in the sense of mode – "most of the five people living in a house is 25, close to 25". He then moved forward with the misinterpretation of average as maximum, and tried to find

three numbers between 12 (or later 13) and 25. In the end, he said the numbers cannot be more than 27 or 28. He tried to generate five numbers from 12 to 25, with the same gaps between two consecutive numbers, starting with a gap of three: 12, 15, 18, 22, 25; he realised that this did not work and then revised and proposed alternatives of 17, 19, 21, 23; then 15, 21, 13, 18, 21 (“three years difference from two consecutive numbers, except for this one, it’s 7 years, right?”) – which was not consistent with the different interpretations of average that he had. As the time was running out, he rushed to the answer, and said to John, “just write something.” The pair ended up with 13, 15, 18, 21, 28, and worked out the relationships between the five people as brothers and sisters. John jumped in to help figure out the relationships between the people based on their ages: “It’s a brother or father or brother or brother or friends?” Arman seemed to have created a mathematical model for the problem as finding five consecutive numbers with equal gaps knowing the minimum and maximum and assign the numbers to ages of people in a family and figure out the relationship. The focus of their attention was on their interpretation of the mathematics requirements. They then worked backwards to reconstruct the context.

This line of reasoning was coded as a *High level of doing mathematics* as the pair engaged in several actions at that level, including:

- Use of complex and non-algorithmic thinking
- Explore and understand the nature of mathematical concepts, processes, or relationships
- Self-monitor and self-regulate their own cognitive processes
- Access relevant knowledge and experiences and make appropriate use of them
- Analyse the task and actively examine the task constraints that may limit possible solution strategies and solutions (Stein & Lane, 1996).

In terms of mathematical practices, we can observe that both students were involved in:

- *Making sense of the problems and persevering in solving them.* Both students started by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analysed givens, constraints, relationships, and goals for the task.
- *Constructing viable arguments and critiquing the reasoning of others.* They understood and use assumptions, definitions (average), and established results in constructing arguments. They justified their conclusions, communicated them to others, and responded to each other’s arguments.
- *Modelling with mathematics.* They applied the mathematics they know to solve problems. They were able to identify important quantities in a practical situation and mapped their relationships using such tools as diagrams. They

routinely interpreted their mathematical results in the context of the situation and reflected on whether the results make sense, possibly improving the model (CCSSI, 2010).

As can be seen, the two frameworks (cognitive demands and mathematical practices) are conceptually disjoint, addressing entirely different aspects of mathematical sophistication with one framework hierarchical in nature and the other one does not assume any particular order. The juxtaposition of the two frameworks informs a more nuanced reading of the data.

DISCUSSION

The paper reports the use of two analytical frameworks to document levels of mathematical sophistications of students' reasoning during collaborative problem solving. The creation of the story lines appears to be useful for tracing the reasoning process of the students. It is a novel approach to apply the cognitive demands framework to document students' mathematical sophistication when reasoning during collaborative problem solving rather than focusing only on task features. The framework appears to be useful for capturing the nuances of the students' reasoning when working on the problem solving task. In addition, the application of the mathematical practices standards as a classificatory framework draws attention to student actions that are valued when solving mathematical problems. The applications of the two analytic frameworks could help advancing ways to examine collaborative problem solving. Furthermore, by applying these two frameworks, the researchers could examine the connections between each level of cognitive demand and the eight mathematical practices. The analysis is descriptive but not explanatory. It represents the first step in a research process directed towards the development of theory in relation to student collaborative problem solving and learning. Using the combined frameworks to identify student pairs or groups engaged in sophisticated mathematical activity, the video and transcript records of their activity can be examined to identify forms of interaction characteristic of such mathematically successful social groups.

The analysis reported here focused on providing an overall evaluation of the reasoning processes of the students. Further analysis is anticipated to examine the finer-grained patterns in mathematical sophistication of the students' reasoning during the pair discussion. Chunking the transcript data into smaller units could reveal patterns of mathematical sophistication evident when the students were negotiating during problem solving. A possibility is to chunk the transcript data into the unit of negotiative events (cf. Clarke, 2001) as a further step to document the levels of mathematical sophistication at this grain size. Furthermore, it could be useful to associate the coding at this grain-size with the coding of other aspects of the student interactions (e.g., student dialogic talk and affect) and use other variables as a way to account for mathematical sophistication. The Social Unit of Learning project concerns the identification of regularities in the negotiative interactions of students and how the social interactions influence the mathematical sophistications of student reasoning during collaborative problem solving. Future work will involve combining the analysis

just described with other analyses of student affect, intersubjectivity, and discursive practice to identify factors of potential value to explain or account for students' mathematical sophistications.

Acknowledgement

This research was supported under Australian Research Council's *Discovery Projects* funding scheme (project number DP170102541).

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