Early language and executive skills predict variations in number and arithmetic skills in children at family-risk of dyslexia and typically developing controls

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A B S T R A C T

Two important foundations for learning are language and executive skills. Data from a longitudinal study tracking the development of 93 children at family-risk of dyslexia and 76 controls was used to investigate the influence of these skills on the development of arithmetic. A two-group longitudinal path model assessed the relationships between language and executive skills at 3–4 years, verbal number skills (counting and number knowledge) and phonological processing skills at 4–5 years, and written arithmetic in primary school.

The same cognitive processes accounted for variability in arithmetic skills in both groups. Early language and executive processes predicted variations in preschool verbal number skills, which in turn, pre-dicted arithmetic skills in school. In contrast, phonological awareness was not a predictor of later arithmetic skills. These results suggest that verbal and executive processes provide the foundation for verbal number skills, which in turn influence the development of formal arithmetic skills. Problems in early language development may explain the comorbidity between reading and mathematics disorder.

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1. Introduction

Reading and mathematics disorder frequently co-occur, but so far we lack an understanding of the cognitive mechanisms that account for this comorbidity. In this paper we explore possible explanations by examining early language and executive skills in children at family-risk of dyslexia. Understanding the influence of these skills on the development of arithmetic is important for understanding the development of, and comorbidities between, language, reading and arithmetic problems. This study focuses on the role of early language skills and domain-general skills of nonverbal IQ and executive skills as foundations for the development of arithmetic skills and the mediating role of the exact verbal number system in typically developing children and children at risk of dyslexia. More specifically, we explore the extent to which nonverbal IQ, language and executive skills may constrain children’s ability to learn to count and to learn number names before the onset of formal teaching. We then relate variations in these domain-specific verbal number skills (counting and number knowledge) to measures of formal arithmetic ability assessed some two years after the onset of formal schooling.

It has often been argued that magnitude processing skills provide the cognitive foundations for the development of arithmetic. Two pre-verbal core systems for representing numerosities have been distinguished. The first is an approximate number system (ANS; Dehaene, 1992) which represents magnitudes in an approximate way. The second system, the object tracking system (OTS), represents small numbers of objects in an exact way (Piazza, 2010). These pre-verbal systems have been considered to be innate (Feigenson, Dehaene, & Spelke, 2004), and to provide the initial basis for representing numerosities and their meaning before language is acquired (von Aster & Shalev, 2007). During the preschool and early school years spoken number—words and Arabic digits are
taught and these verbal-symbolic representations acquire their meaning by being mapped onto the preverbal core systems.

According to van Aster and Shalev (2007) the acquisition of the verbal-symbolic number system not only builds on the preverbal core systems but also depends on language and domain-general skills. For example, if language or executive skills are deficient, then verbal number skills (e.g., counting) might not develop normally and associations between verbal-symbolic representations and non-symbolic representations would suffer. Consequently, children with language or reading problems are at risk of developing arithmetic difficulties.

Indeed, previous studies have found that individuals with language or reading problems perform poorly on arithmetic tasks (e.g., fact retrieval) compared to individuals without language or reading problems (De Snedt & Boets, 2010; Göbel & Snowling, 2010; Miles, Haslam, & Wheeler, 2001; Simmons & Singleton, 2006). Furthermore, individuals with reading disorder are impaired in verbal number tasks, such as counting, but seem to be unimpaired on nonverbal tasks tapping the ANS (e.g., Göbel & Snowling, 2010; Moll, Göbel, & Snowling, 2014). It has been argued, that problems in fact retrieval in children with dyslexia are mediated by their reading problems (Mammarella et al., 2013). Thus, poor arithmetic skills in children with reading disorder are associated with problems in language processing, rather than with problems in basic number processing. In contrast, children with dyscalculia seem to be impaired in a wider range of number skills and their poor arithmetic skills are associated with problems in basic number processing (Moll et al., 2014). Consistent with this, the core deficits underlying reading disorder are distinct from those underlying mathematics disorder (Ashkenazi, Black, Abrams, Hoef, & Menon, 2013; Landerl, Fussenerger, Moll, & Willburger, 2009), with a phonological deficit being one proximal cause of reading difficulties (Vellutino, Fletcher, Snowling, & Scanlon, 2004) and a deficit in processing numerosities being associated with mathematics difficulties (Butterworth, 2010; Wilson & Dehaene, 2007). It follows that early language problems and non-verbal number deficits may constitute separable risk factors for arithmetic disorder. Children with poor language skills may therefore be at risk of developing arithmetic difficulties in spite of the fact that their non-verbal number skills are unaffected.

In order to trace possible causal influences from early cognitive skills to later arithmetic attainment, longitudinal studies starting before children enter formal education are required. Remarkably few such studies have been published to date. One of the few studies (LeFevre et al., 2010) followed children from the age of 4½ to 7½ years. At the beginning of the study, language measures (vocabulary and phoneme deletion) predicted concurrent variations in children's number naming, whereas a measure of quantitative knowledge, namely subitizing (immediate apprehension of small quantities, as measured by the time used to indicate the number of dots in an array of 1–3 dots), predicted concurrent variations in a nonverbal arithmetic task. Variations in a “linguistic” factor (vocabulary, phoneme awareness and number naming) and a “quantitative” factor (subitizing and nonverbal arithmetic) were both predictors of conventional arithmetic skills assessed when children were 7½ years old, with effects from the “linguistic” factor being stronger. These findings suggest that variations in children's early language skills (broadly defined) provide a foundation for the development of arithmetic skills once formal teaching begins. The role of language skills in the development of mathematical abilities is further supported by studies showing that children with reading impairment perform poorly on arithmetic compared to typically developing controls (Donlan, Cowan, Newton, & Lloyd, 2007; Fazio, 1996; Koponen, Mononen, Rasanen, & Ahonen, 2006).

In addition to the role of language skills, studies analyzing the impact of domain-general skills on arithmetic have reported associations between aspects of executive functioning, but not nonverbal IQ, and children's mathematical skills (e.g., Bull, Espy, & Wiebe, 2008; Bull & Scerif, 2001; Espy et al., 2004; van der Sluis, de Jong, & van der Leij, 2004). However, the importance of nonverbal IQ may increase across grades (Geary, 2011) when the demands of arithmetic tasks increase (e.g., written calculations with two-digit numbers) and when tasks involve problem solving (Hembree, 1992; Vickers, Mayo, Heitmann, Lee, & Hughes, 2004; Xin & Zhang, 2009). In line with this, Kyttälä and Lehto (2008) reported that nonverbal IQ predicted individual differences in complex mental arithmetic tasks and written word problems (including percentage calculations and equations) in 15–16 year olds (see also Deary, Strand, Smith, & Fernandes, 2007). Nonverbal IQ is also strongly related to nonverbal number tasks (e.g., relations between quantities, number line estimation). For example, Hornung, Schlitz, Brunner, and Martin (2014) recently reported a strong and direct association between nonverbal IQ (assessed at the end of kindergarten) and performance in a number line estimation task assessed one year later; this association was not mediated by preschool number skills.

Turning to executive functioning, comparatively little is known about the development of executive functions in the preschool years and their relationship to later academic skills and studies assessing executive functions in children as young as 3–4 years are very rare (see Garon, Bryson, & Smith, 2008). In the influential framework of Miyake et al. (2000) executive functions form a unitary construct, but with partly dissociable components: (1) working memory/updating (holding, manipulating and updating information in mind) (2) inhibition (the ability to suppress irrelevant or distracting information and prevent predominant responses), and (3) set shifting (the flexibility to switch between different tasks).

The relationship between executive functioning and the development of mathematical skills was recently summarized in a review by Cragg and Gilmore (2014). For working memory it has been suggested that the ability to manipulate and update information may be particularly crucial for developing mathematical skills. Such an ability is required when solving word problems as well as when calculating two-digit additions involving carrying (e.g., 15 + 17). It has been suggested that the role of verbal working memory increases with age, when the problems being solved become more complex (Geary, 2011; McKenzie, Bull, & Gray, 2003; von Aster & Shalev, 2007). Inhibition has been linked to performance in tasks including inversion shortcuts in addition and subtraction (Robinson & Dubé, 2013) as well as in counting tasks when counting on from the larger addend instead of counting on from the first addend (e.g. Cragg & Gilmore, 2014) and in multiplication tasks when suppressing answers to related but incorrect number facts (see also Bull & Scerif, 2001; Espy et al., 2004). In a study by Blair and Razza (2007) inhibitory control was significantly related to early mathematical ability. Finally, the ability to switch between tasks (set shifting) is required when alternating between different operations (e.g., adding and subtracting), for example when solving complex mathematical problems. Set shifting is not acquired until the end of the preschool period and is supposed to be more important later in development (Bull et al., 2008) for learning new concepts and procedures (Cragg & Gilmore, 2014).

However, it should be noted that studies in young children often fail to find the componential structure of executive functions (e.g., Wiebe, Espy, & Charak, 2008), suggesting a more unitary structure of executive functions during childhood. Nonetheless, the ability to deal with conflict during information processing together with the ability to focus attention and ignore irrelevant information...
(selective attention and inhibition) appear to be critical aspects of the development of executive functions in the preschool years (Garon et al., 2008) and are necessary during the execution of arithmetical operations (Blair & Razza, 2007; Szücs, Devine, Soltesz, Nobes, & Gabriel, 2014).

Thus, language and executive skills may place constraints on the development of arithmetic skills; however the mediating mechanisms have yet to be determined. Here we report the findings of a study to examine the cognitive precursors of arithmetic skills in children at high risk of learning disorders (children at family risk of dyslexia). Given the frequent comorbidity of reading and mathematics disorder, this approach is likely to yield important information regarding the early predictors of individual differences in arithmetic skills.

One mechanism by which early language and executive skills may affect later arithmetic skills is through their influence on how children learn the names of number symbols (von Aster et al., 2007; Krajewski et al., 2009), or their ability to match Arabic numerals to number–word sequence in kindergarten predict basic arithmetic skills in school (Aunio & Niemivirta, 2010; Aunola, Leskinen, Lerkkanen, & Nurmi, 2004; Koponen, Salmi, Eklund, & Aro, 2013; Passolunghi, Vercelloni, & Schadee, 2007; Stock, Desoete, & Roeyers, 2009) and Grade 1 counting skills have been shown to predict arithmetic at the age of 10 (Koponen et al., 2013). Second, early number skills predict later mathematics achievement (e.g., Geary, 2011; Jordan, Kaplan, Locuniak, & Ramineni, 2007; Jordan, Kaplan, Ramineni, & Locuniak, 2009) though the unique contribution of number knowledge (as defined by the ability to match Arabic numerals to their verbal labels) is uncertain because extant studies have used different measures of early number skills.

In addition, several studies claim that individual differences in arithmetic are predicted by phonological awareness (De Smedt & Boets, 2010; De Smedt, Taylor, Archibald, & Ansari, 2010; Fuchs et al., 2005, 2006; Geary & Hoard, 2001; Hecht, Torgesen, Wagner, & Rashotte, 2001; Leather & Henry, 1994; Simmons, Singleton, & Horne, 2008) and that children with poor phonological skills are more likely to develop mathematical difficulties (Jordan, Wylie, & Mulhern, 2009) and that children with poor phonological awareness and arithmetic skills is not consistently found (Duran, Holune, Larkin, & Snowling, 2005; de Jong & van der Leij, 1999; Passolunghi & Lanfranchi, 2012; Passolunghi, Mammarella, & Althe, 2008). Moreover, the mechanisms which could account for the relationship between phonological awareness and arithmetic skills are unclear.

In summary, previous research suggests that variations in preschool abilities, especially in language and executive functions, influence the development of domain-specific precursors of arithmetic such as counting skills and number knowledge. Counting and number knowledge, in turn, play an important role in learning formal arithmetic once schooling begins. In the current study, we investigated possible causal pathways from early domain-general abilities (nonverbal IQ, and executive functions) and language skills to preschool number and phonological processing skills which may, in turn, influence the development of arithmetic skills in primary school. In addition, we compared the pattern of predictive influences between typically developing children and children at risk of dyslexia. Children at risk of dyslexia are characterized by poorer language and executive skills which are likely causal risk factors for developing later arithmetic problems. If the patterns of predictors of arithmetic development differ between groups, this could have important implications for early interventions to circumvent arithmetic difficulties.

2. Method

2.1. Participants

The data reported here form part of a large longitudinal study (The Wellcome Language and Reading Project). Families with 3- and 5-year-old children were recruited to the study via advertisements and speech and language therapy services. For the purpose of the current study, children were classified into two groups: a group of children at family-risk of dyslexia (FR) and a typically developing control group (TD). None of the children included in the current analysis had specific language impairment. Language impairment status was determined using three subtests (Basic Concepts, Expressive Vocabulary, and Sentence Structure) of the Clinical Evaluation of Language Fundamentals (CELF-Preschool 2 UK; Semel, Wig, & Secord, 2006) and the screener from the Test of Early Grammatical Impairment (TEGI; Rice & Wexler, 2001). The criterion for specific language impairment was a standard score at least one standard deviation below the mean on either at least two out of the three standardized language measures or on at least one together with failure on the screener.

Children were classified as FR when a parent self-reported to be dyslexic using the Adult Reading Questionnaire (ARQ; Snowling, Dawes, Nash, & Hulme, 2012) or based on objective testing (standard score < 90 on a composite of nonword reading and spelling or standard score < 96 and a discrepancy between nonverbal IQ and literacy of 1.5 standard deviations) of parents who consented. Children were also considered at FR if an older sibling had a diagnosis of dyslexia (Nash, Hulme, Gooch, & Snowling, 2013 for details). The sample consisted of 169 children (76 TD and 93 FR). None of the children met any of our exclusionary criteria (chronic illness, deafness, neurological disorder, English as 2nd language, care provision by local authority). The drop-out rate (4%) was small (see Section 3.2 for the method of handling of missing data).

All children came from British white families in the county of North Yorkshire, England. Socioeconomic status (SES) was calculated using the UK Indices of Multiple Deprivation (Department of Communities and Local Government, 2010). The Index provides a relative rank according to deprivation value using postcodes. At recruitment, the current sample showed a relatively high SES score indicating low levels deprivation with a mean percentage rank of 66% (FR = 61% and TD = 72%).

Ethical clearance for the study was granted by the University of York, Department of Psychology's Ethics Committee and the NHS Research Ethics Committee. Parents provided informed consent for their child to participate.

2.2. Design

For the purpose of the current investigation children’s data were drawn from three assessment phases (t1, t2, t3), when aged 3–4, 4–5 and 5–7 years old respectively. At t1, we focused on cognitive foundations which might have an impact on precursors of arithmetic skills (nonverbal IQ, executive functioning, and broader oral language skills). At t2, exact verbal-symbolic number skills (counting and number knowledge) and phonological skills were assessed. Finally, at t3 the outcome measure (arithmetical skills) was assessed.
2.3. Tests and procedures

2.3.1. Tests at t1

2.3.1.1. Nonverbal IQ. Nonverbal IQ was measured using two sub-tests from the Wechsler Preschool and Primary Scale of Intelligence (current version at t1: WPPSI-III; Wechsler, 2003): Block Design and Object Assembly. A composite score was calculated based on the mean of z-standardized scores for the two subtests.

2.3.1.2. Executive functioning. Three subtests, which have been proved to be suitable for children at this age and to show moderate stability over time (Gooch, Hulme, Nash, & Snowling, 2014) were administered to assess complex response inhibition (Dog-Bird Go/No-Go task and the Head Toes Knees and Shoulders (HTKS) task) and selective attention (Visual Search task); these constructs represent important components of executive functioning in the preschool years (Garon et al., 2008) and are believed to play an important role in the early phases of arithmetic development (Cragg & Gilmore, 2014).

Go/No-Go task. A version of the Bear-Dragon Go/No-Go task (Reed, Pien, & Rothbart, 1984) was administered; the child has to follow instructions and make a wave, thumbs up, point, or a fist. The child is asked to follow the instructions only when the nice puppet (dog) asks, but not when the naughty puppet (bird) asks. The 16 test trials are administered after 4 practice trials and are scored as 0 (incorrect) or 1 (correct). An efficiency score was calculated: number of hits (responses to dog)/total responses (responses to dog plus responses to bird).

Heads-Toes-Knees-and Shoulders (HTKS) task (Burrage et al., 2008). The child is asked to do the opposite of what the examiner says. When the examiner says “touch your head”, the child’s correct reaction is to touch toes and vice versa; when the examiner says “touch your knees”, the correct reaction is to touch shoulders. Following practice, for the first 10 items (part 1) only two prompts (head and toes) are used; for the following 10 items (part 2) all four prompts are included. Part 2 is only administered if the child responds correctly to at least 5 items in part 1. Each item is scored 0 (incorrect), 1 (self-corrected response), or 2 (correct); maximum score = 40. Interrater-reliability is reported to be high (alpha = .98; Ponitz et al., 2008). Stability over one year was .51 (Gooch et al., 2014).

Visual Search task (Apples Task; Breckenridge, 2008). A picture showing red apples (targets), red strawberries and white apples (distractors) is presented and the child’s task is to find as many red apples as possible within 1 min. Omission and commission errors were recorded and an efficiency score was calculated based on the number of targets correct – commission errors/60. Stability over one year was .53 (Gooch et al., 2014).

Given the unitary structure of executive functions during childhood, the three measures of executive functioning were combined into a composite score based on the mean of the z-standardized scores for the three subtests (see Rothlisberger, Neuenschwander, Cimeli, & Roebers, 2013).

2.3.1.3. Language skills. Two subtests from the current version of a standardized language test, the Clinical Evaluation of Language Fundamentals (CELF-Preschool 2 UK; Semel et al., 2006) measured oral language skills.

Expressive Vocabulary. The child names objects of increasing difficulty (e.g., carrot, telescope) or describes what a person is doing (e.g., riding a bike). According to the test manual Cronbach’s Alpha = .82 for this age group.

Sentence Structure. The child hears a sentence and selects from a choice of four, the picture it refers to. The subtest is measuring the child’s understanding of grammatical rules at the sentence level. According to the test manual Cronbach’s Alpha = .78 for this age group.

A composite language score was calculated based on the mean of the z-standardized scores for the two subtests.

2.3.2. Tests at t2

2.3.2.1. Counting. Two tests were developed in order to measure counting skills.

Dot counting: This test comprises 8 pictures with dots (3, 4, 7, 6, 15, 12, 14, 11) and the child was asked to count how many dots are presented on each picture and provide the cardinal value of the number of dots. The responses are scored as 0 (incorrect) or 1 (correct); maximum score = 8. Guttmans split half coefficient calculated based on 86 children at t2 was .64.

Counting maximum task: The child had to count aloud as far as possible. The score was the highest number the child was able to count to without making any errors. Object/dot counting and counting on tasks have been used in test batteries (e.g., TEDI-Math; Van Nieuwenhoven, Greogire, & Noel, 2001) and in several previous studies (e.g., Donlan et al., 2007; Krajewski & Schneider, 2009a).

A composite score for counting was calculated based on the mean of the z-standardized scores for the two counting subtests.

2.3.2.2. Number knowledge. Number knowledge was assessed using a number recognition and a number writing task; similar tasks have been used in previous studies (e.g., Göbel, Watson, Lervåg, & Hulme, 2014; Moll et al., 2014). Cronbach’s Alpha = .90.

Number recognition: The child had to identify a written numeral. There were 13 items with numerals ranging between 0 and 100.

Number writing: The child was asked to write 6 numerals (0, 3, 4, 7, 6, 10).

A composite score for number knowledge was calculated based on the mean percentages correct for the two subtests.

2.3.2.3. Phonological awareness. Three subtests were administered that have been proved to be suitable for children at this age (see Carroll, Snowling, Stevenson, & Hulme, 2003): syllable matching, alliteration matching and phoneme isolation. Cronbach’s Alpha = .92.

Syllable matching: The child heard a word and had to indicate which of two other words sounds the same at the beginning (6 items) or at the end (6 items), respectively (e.g., fireworks: doctor or fireman).

Alliteration matching (10 items): The child had to identify which of two words starts with the same sound as a target word (e.g., pot: duck or peach).

Phoneme isolation: The child had to identify the first (8 items) or last sound (8 items) of a given nonword (e.g., first sound of /gulf/).

A composite score for phonological awareness was calculated based on the mean percentages correct for the three subtests.

2.3.3. Tests at t3

2.3.3.1. Arithmetic. To measure arithmetic skills, written timed addition and subtraction tests were administered. Stability of the test over one year in the current sample is high (r = .76, p < .001). Children were instructed to complete as many single digit additions/subtractions as possible within 1 min (max = 30 per subtest). Each subtest was presented on an A4 sheet and children were asked to write down the answer. All operands and answers were below 20. Items 1 to 20 include only single digits as operands and answers (e.g., addition: 2 + 5; subtraction: 7 – 3); items 21 to 30 involve crossing the decade (e.g., addition: 5 + 7; subtraction: 14 – 6). The number of correctly solved items per second (efficiency) was calculated for each subtest. The efficiency score is a measure of fact
retrieval; children who still rely on counting strategies will solve fewer items per second compared to those who are able to directly access arithmetic facts.

A composite score for arithmetic was calculated based on the mean of the efficiency score for the two subtests.

2.3.4. Assessment

At all three time-points children were assessed individually. Assessments at t1 and t2 took place in the family’s homes; assessments at t3 took place in a quiet room at school. The tests were administered in a fixed order by trained members of the project team.

3. Results

3.1. Descriptive statistics and correlations

Descriptive statistics for all individual measures are reported in Table 1, separately for the group at family-risk of dyslexia (FR) and for the typically developing group (TD).

There was a small but fairly consistent trend for the FR group to do more poorly than the TD group. Initial analyses thus explored the relationships between key variables in each group separately. However, there was no significant difference in the slopes of the regression functions for the two groups relating the key predictors to later arithmetic scores. The correlations between key variables are shown separately for the two groups in Table 2. In general, correlations were comparable between the two groups. In both groups, the strongest correlations with later arithmetic skills were implied by this model by conducting a Confirmatory Factor Analysis in which there were 5 correlated factors (Executive Functions, Language Skills, Counting, Number Knowledge, Phonological Awareness and Arithmetic Efficiency). The model fitted the data adequately, $\chi^2 (62) = 82.965, p = .039$, Root Mean Square Error of Approximation (RMSEA) = .045 (90% CI = .011–.068), Comparative Fit Index (CFI) = .97, confirming that the structure of the underlying abilities specified in the path model was justified.

We assessed the longitudinal relationships between key measures using the path model shown in Fig. 1. This model is based on composite scores derived by averaging z-scores for groups of variables. Correlations between all measures are reported in Table 3. Prior to testing the model in Fig. 1 we checked the factor structure implied by this model by conducting a Confirmatory Factor Analysis in which there were 5 correlated factors (Executive Functions, Language Skills, Counting, Number Knowledge, Phonological Awareness and Arithmetic Efficiency). The model fitted the data adequately, $\chi^2 (57) = 75.778, p = .09$, RMSEA = .052 (90% CI = .019–.074), CFI = .96, confirming that the structure of the underlying abilities specified in the path model was justified.

3.2. Multi-group path model

A longitudinal multi-group (FR and TD groups) path model with observed variables was estimated with Mplus 7.3 (Muthén & Muthén, 1998–2014). There were very small amounts of missing data (only 7/169 children dropped out; and 7/169 children were absent for the first assessment). All analyses handled missing data using Full Information Maximum Likelihood estimation (the default in Mplus; Muthén & Muthén, 1998–2014). We assessed the assumption underlying the use of FIML that data were Missing Completely at Random using Little’s MCAR test, which was not significant ($\chi^2 (11) = 5.2943, p = .17$). To allow for deviations from normality in some variables, and to obtain robust standard errors for the indirect effects in the model, we used bias-corrected bootstrapped standard errors (Preacher & Hayes, 2008). A preliminary model included nonverbal IQ at t1 as a domain-general predictor of all t2 measures (counting skills, number knowledge and phonological awareness) but because nonverbal IQ did not have any direct

Table 1
Means (standard deviations) and t-values for measures assessed at all three time points for the FR and the TD group.

<table>
<thead>
<tr>
<th>Variables</th>
<th>FR</th>
<th>TD</th>
<th>t</th>
<th>Cohen’s $d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>93</td>
<td>76</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gender (male/female)</td>
<td>54/39</td>
<td>37/39</td>
<td></td>
<td></td>
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T1

<table>
<thead>
<tr>
<th>Variables</th>
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<th>TD</th>
<th>t</th>
<th>Cohen’s $d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age in months</td>
<td>45.34 (3.64)</td>
<td>44.69 (3.20)</td>
<td>1.17</td>
<td>-.19</td>
</tr>
<tr>
<td>IQ: block design$^a$</td>
<td>19.12 (3.38)</td>
<td>20.63 (2.94)</td>
<td>2.97**</td>
<td>.47</td>
</tr>
<tr>
<td>IQ: object assembly$^b$</td>
<td>18.82 (7.50)</td>
<td>21.68 (7.24)</td>
<td>2.41*</td>
<td>.39</td>
</tr>
<tr>
<td>Executive functions: go/no-go$^c$</td>
<td>.75 (.23)</td>
<td>.81 (.22)</td>
<td>1.36</td>
<td>.27</td>
</tr>
<tr>
<td>Executive functions: HTKS$^d$</td>
<td>8.46 (10.18)</td>
<td>13.39 (12.61)</td>
<td>2.45*</td>
<td>.43</td>
</tr>
<tr>
<td>Executive functions: visual search$^e$</td>
<td>.11 (.06)</td>
<td>.13 (.06)</td>
<td>1.41</td>
<td>.33</td>
</tr>
<tr>
<td>Language: vocabulary$^f$</td>
<td>18.53 (5.32)</td>
<td>20.69 (5.24)</td>
<td>2.55*</td>
<td>.41</td>
</tr>
<tr>
<td>Language: sentence structure$^g$</td>
<td>13.52 (2.77)</td>
<td>14.39 (3.27)</td>
<td>1.81</td>
<td>.29</td>
</tr>
</tbody>
</table>

T2

<table>
<thead>
<tr>
<th>Variables</th>
<th>FR</th>
<th>TD</th>
<th>t</th>
<th>Cohen’s $d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age in months</td>
<td>57.03 (4.23)</td>
<td>55.69 (3.43)</td>
<td>2.21*</td>
<td>-.35</td>
</tr>
<tr>
<td>Counting: dots$^h$</td>
<td>4.75 (1.68)</td>
<td>5.22 (1.35)</td>
<td>1.94</td>
<td>.31</td>
</tr>
<tr>
<td>Counting: maximum$^i$</td>
<td>24.77 (13.60)</td>
<td>28.86 (14.89)</td>
<td>1.82</td>
<td>.29</td>
</tr>
<tr>
<td>Number knowledge: recognition$^j$</td>
<td>58.12 (24.08)</td>
<td>65.28 (23.79)</td>
<td>1.91</td>
<td>.30</td>
</tr>
<tr>
<td>Number knowledge: writing$^k$</td>
<td>55.24 (29.15)</td>
<td>54.28 (28.14)</td>
<td>.21</td>
<td>-.03</td>
</tr>
<tr>
<td>PA: syllable matching$^l$</td>
<td>80.34 (16.73)</td>
<td>83.45 (11.66)</td>
<td>1.39</td>
<td>.22</td>
</tr>
<tr>
<td>PA: alliteration matching$^m$</td>
<td>74.77 (20.57)</td>
<td>83.42 (19.38)</td>
<td>2.73**</td>
<td>.43</td>
</tr>
<tr>
<td>PA: phoneme isolation$^n$</td>
<td>52.42 (32.44)</td>
<td>58.27 (31.99)</td>
<td>1.09</td>
<td>.18</td>
</tr>
</tbody>
</table>

T3

<table>
<thead>
<tr>
<th>Variables</th>
<th>FR</th>
<th>TD</th>
<th>t</th>
<th>Cohen’s $d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age in months</td>
<td>78.94 (4.62)</td>
<td>78.96 (3.97)</td>
<td>.03</td>
<td>.00</td>
</tr>
<tr>
<td>One-minute addition$^o$</td>
<td>7.99 (4.65)</td>
<td>10.26 (5.07)</td>
<td>2.95**</td>
<td>.47</td>
</tr>
<tr>
<td>One-minute subtraction$^p$</td>
<td>5.71 (3.69)</td>
<td>7.32 (3.49)</td>
<td>2.83**</td>
<td>.45</td>
</tr>
</tbody>
</table>

PA: phonological awareness.

$^a$ p < .05; $^b$ p < .01.

$^a$ Raw score.

$^b$ Efficiency score.

$^c$ Percent correct.
or indirect effect on arithmetic skills, it was dropped from the model.

In the final model, shown in Fig. 1, executive functions and language skills at t1 (age 3; 9 years) predict counting skills, number knowledge and phonological awareness at t2 (age 4; 8 years) and counting and number knowledge at t2, in turn, predict variations in formal arithmetic skills at t3 (age 6; 7 years). In this model all unstandardized path weights and all covariances were constrained to be equal across the two groups. The model gives an excellent fit to the data, $\chi^2 (17) = 14.182$, $p = .654$, Root Mean Square Error of Approximation (RMSEA) = .000 (90% CI = .000-.082), Comparative Fit Index (CFI) = 1.0, Standardized Root Mean Square Residual (SRMR) = .068, with no significant difference in fit between the constrained model shown and an unconstrained model in which all parameters were free to vary between groups ($\Delta\chi^2 (11) = 6.359$, $p = .848$). The estimates shown are standardized values which differ slightly between groups due to differences in variances.

A number of features of the model are noteworthy. First, executive functions and language at t1 both predict variations in the two domain-specific precursors of formal arithmetic skills (counting and number knowledge) assessed at t2. In addition, language but not executive skills at t1, predicts variations in phonological awareness at t2. Finally, both counting skills and number knowledge at t2 (but not phonological awareness) predict formal arithmetic skills at t3. These two variables together account for 46% (FR) and 40% (TD) of the variance in arithmetic at t3.

In this model there are four indirect effects relating early variations in executive function and language skills at t1 to later variations in arithmetic at t3. These are:

1. From executive functions at t1 to counting skills at t2, which in turn predict formal arithmetic skills at t3.
2. From language at t1 to number knowledge at t2, which in turn predict formal arithmetic skills at t3.
3. From language at t1 to phonological awareness at t2, which in turn predict formal arithmetic skills at t3.
4. From counting skills at t2 to number knowledge at t2, which in turn predict formal arithmetic skills at t3.

These four indirect effects together account for 46% (FR) and 40% (TD) of the variance in arithmetic at t3.
variations in arithmetic skills at t3. Three of these indirect effects are statistically reliable in both groups: (1) language \(\rightarrow\) counting \(\rightarrow\) arithmetic: FR-group standardized effect = .090; \(p = .019;\) TD-group standardized effect = .082; \(p = .013;\) (2) executive function \(\rightarrow\) number knowledge \(\rightarrow\) arithmetic: FR-group standardized effect = .075; \(p = .021;\) TD-group standardized effect = .080; \(p = .022;\) (3) language \(\rightarrow\) number knowledge \(\rightarrow\) arithmetic: FR-group standardized effect = .092; \(p = .029;\) TD-group standardized effect = .083; \(p = .044.\) The last indirect effect, though of very similar magnitude in both groups, was only significant in the TD-group: (4) executive function \(\rightarrow\) counting \(\rightarrow\) arithmetic: FR-group standardized effect = .055; \(p = .056;\) TD-group standardized effect = .059; \(p = .048.\) It should be noted that this model shows complete mediation. Although language and executive function at t1 are significantly correlated with arithmetic skills in both groups the direct effects of these variables on later arithmetic skill (executive function \(\rightarrow\) arithmetic; language \(\rightarrow\) arithmetic) do not approach significance in either group after the mediated relationships have been accounted for (TD group: executive function \(\rightarrow\) arithmetic standardized effect = .075; \(p = .388;\) language \(\rightarrow\) arithmetic standardized effect = .091; \(p = .627;\) FR group: executive function \(\rightarrow\) arithmetic standardized effect = .070; \(p = .388;\) language \(\rightarrow\) arithmetic standardized effect = .101; \(p = .627).\)

4. Discussion

The present study aimed to clarify the cognitive foundations of formal arithmetic by investigating putative causal relationships between early language and domain-general cognitive skills (nonverbal IQ, and executive functions), preschool exact verbal number skills (counting and number knowledge) and phonological processing skills, and written arithmetic assessed in primary school. Furthermore, we investigated whether the predictors of arithmetic skills differ between children at risk of dyslexia and typically developing children.

Our findings indicate that children at risk of dyslexia performed more poorly on the domain-general cognitive foundations of arithmetic as well as on the outcome arithmetic measure. However, the same cognitive processes accounted for variability in arithmetic skills in the two groups.

With respect to the predictors of arithmetic development, the results of the study provide both confirmation and disconfirmation of existing hypotheses. The main novel finding, consistent with research reporting longitudinal relationships between preschool number skills and later formal arithmetic abilities (Aunio & Niemivirta, 2010; Aunio et al., 2004; Koponen et al., 2013; Öster gren & Träff, 2013; Passolunghi et al., 2007; Stock et al., 2009) is that arithmetic skills in primary school are directly predicted by preschool verbal number skills (number knowledge and counting), and these skills, in turn, are influenced by earlier variations in oral language skills. Our results extend previous findings by showing that counting and number knowledge are equally strong unique predictors of individual differences in later arithmetic, accounting together for 40–46% of variance in these skills.

In contrast, our findings fail to support the idea that phonological awareness exerts a causal influence on the development of arithmetic skills, at least when measured in 4–5 year-olds, despite the fact that oral language skills at t1 predicted phonological awareness at t2. A proviso is that De Smedt and colleagues (De Smedt & Boets, 2010; De Smedt et al., 2010) argued for a specific relationship between phonological awareness and measures of arithmetic requiring fact retrieval. Although, combining addition and subtraction in the current model might have masked this relationship, additional analyses examining either addition or subtraction efficiency as separate outcome measures revealed no differences in the predictive patterns. We therefore propose that verbal number processes concerned with learning the count sequence and the ability to learn Arabic numerals and map them onto verbal codes, are both critical skills for the development of basic arithmetic abilities but phonological awareness is not.

The second main finding of the current study is that executive skills at t1, but not nonverbal IQ, predicted counting and number knowledge at t2. Consistent with this, previous research reports associations between measures of executive functions with mathematics achievement in school (Bull et al., 2008; Bull & Sieroff, 2001; van der Sluis et al., 2004) and with preschool number skills (Espy et al., 2004; Passolunghi & Lanfranchi, 2012). Our measures of executive skill included a measure of selective attention and two measures requiring complex response inhibition. Complex response inhibition tasks involve memory skills, such as holding a rule in mind and inhibition/self-regulation when inhibiting a prepotent response in order to respond to the rule. Arguably both developing one-to-one correspondence and learning to count proficiently depend heavily upon these skills. More specifically, children need the ability to monitor linguistic processes and the ability to suppress less sophisticated counting strategies in order to
use more efficient ones (e.g., counting on from the larger addend instead of counting on from the first addend; Cragg & Gilmore, 2014). Given the componential structure of executive skills in later developmental stages, future studies, analyzing the role of these skills in school-aged children, might be able to distinguish between different component skills (including complex working memory tasks) in order to measure the association between executive component skills and different aspects of mathematics.

In contrast to language and executive skills, nonverbal IQ did not explain significant variation in early number skills. This is somewhat surprising given that both our IQ and arithmetic measures involve processing speed. However the role of processing speed (and nonverbal IQ as measured by tasks including speed components) in arithmetic fluency may become more important later in development once basic calculation skills are acquired and the speed of solving calculations then explains a significant amount of individual differences in arithmetic.

In addition, we cannot exclude the possibility that nonverbal IQ becomes more important when task demands increase (Geary, 2011) or that it plays a more important role in aspects of mathematics beyond arithmetic which were not tested in the current study (e.g. problem solving). It is clearly possible that other components of mathematical abilities are influenced by different cognitive skills (Dowker, 2005a, 2008; Jordan, Mulhern, & Wylie, 2009).

It should be noted that we did not include measures of nonverbal number skills, such as estimation or magnitude comparison tasks. This was outside the scope of the current study as our main interest was in analyzing the impact of language and domain-general skills on preschool verbal number skills in a sample of children at risk of dyslexia, characterized by language delays, and poorer executive skills compared to typically developing controls. However, future studies including a wider range of number skills could further specify the longitudinal relationship between nonverbal and verbal number tasks and the association with later arithmetic skills in children at risk of dyslexia.

In summary, the developmental pathway from language and executive functions in preschool to formal arithmetic in the early school years is mediated by counting and number knowledge but early phonological awareness does not play a role. Why are our findings different from those reported previously? Phonological awareness, like counting and number knowledge, was predicted by oral language but it was not predicted by executive skills. We argue that the shared variance between phonological awareness and preschool number skills is attributable to broader language abilities which underpin their development. We can speculate that, if oral language is not controlled in studies of the development of arithmetic, then a measure of phonological awareness will act as a proxy measure for these skills. Our findings undermine the importance of testing causal theories using models that include both direct and mediated effects. Our longitudinal results extend previous concurrent findings (Vukovic & Lesaux, 2013) showing an indirect relationship between language skills and arithmetic knowledge mediated by symbolic number reasoning.

To conclude, our findings clarify possible causal influences on the development of early arithmetic skills. Language and executive skills at 3–4 years of age constrain the ability to learn to count and to match Arabic numerals to their verbal labels one year later and exert an indirect influence on arithmetic ability via these more proximal domain-specific precursors. The problems in early language development frequently reported in children with dyslexia therefore constitute a risk factor for later arithmetic, as well as reading, difficulties. Moreover, such early language difficulties might help explain the common co-morbidity of reading and arithmetic disorder.

Our findings also have practical implications: while educators are well aware that children with a family history of dyslexia are at risk of developing literacy difficulties, less is known about the association between language and arithmetic skills and the increased risk for developing arithmetic problems in this group. Therefore early intervention programs generally focus on precursors underlying literacy skills (i.e., phonological awareness skills), while interventions addressing verbal number skills are often omitted. Given that counting and number knowledge mediate the association between language and arithmetic skills, intervention programs targeting these skills (Dowker, 2005b; Praet & Desoete, 2014) are likely to be helpful for improving number skills in preschool children at risk for dyslexia.

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References


