Young Indigenous Students' Engagement with Growing Pattern Tasks: A Semiotic Perspective

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The aim of this study was to determine the role of semiotics in assisting young Indigenous students to engage with and identify the general structure of growing patterns. The theoretical perspective of semiotics underpinned the study. Data are drawn from two Year 3 students, including analysis of pretest questions and two conjecture-driven lessons. Results indicate that particular semiotic signs (iconic signs) contribute to how young Indigenous students attend to, and identify the structure of growing patterns.

In Indigenous contexts a deficit perspective with regard to students' capability to learn persists, and impacts on the types of mathematics they experience in their classrooms (Warren, Cooper, & Baturo, 2010). Teachers continue to hold "low expectations for Indigenous students and perceptions that the gap in educational outcomes… [are] somehow normal" (Purdie et al., 2011, p.4). Thus it is conjectured that in these contexts there are limited opportunities for Indigenous students to engage in higher levels of mathematical thinking, such as generalising mathematical structures. The ability to generalise is an important aspect of algebraic thinking, and is the key to success in higher levels of mathematics (Lee, 1996). The development of this ability occurs as early as Year 2, and leads to a deeper understanding of mathematical structures (Blanton & Kaput, 2011; Cooper & Warren, 2011). In addition, the exploration of numberless situations, such as growing patterns provides an opportunity for young students to engage in powerful schemes of thinking (Carpenter, Franke and Levi, 2003).

Past research indicates that young non-Indigenous students are capable of generalising growing patterns (Blanton & Kaput, 2005; Leung, Krauthausen & Rivera, 2012; Warren, 2005), however, little is known about (a) Indigenous students' capability to generalise growing patterns, and (b) what types of tasks help Indigenous students to generalise. The focus of this paper is to explore how pattern task development, and the use of semiotics can enhance and support the engagement of young Indigenous students' in early algebraic thinking. In particular, the research question, how does semiotics assist young Australian Indigenous students to engage with and identify the general structure of growing patterns?

Literature

Growing patterns are characterised by the relationship between elements, which increase or decrease by a constant difference. In developing understanding of a growing pattern structure, students are asked to form the functional relationship between the terms in the pattern and their position. That is, they are asked to reconsider growing patterns as functions (covariational thinking – the generalisable relationship between the term and its position), rather than as a variation of one data set (recursive thinking – relationship between successive terms within the pattern itself) (Warren, 2005).

Findings from past research indicates that the way in which growing patterns are presented and taught potentially limits students' awareness and accessibility to the generalisable structure of the pattern (Küchemann, 2010; Moss & McNab, 2011). Often the growing patterns presented to students are abstract representations, displayed as drawn

geometric shapes (e.g., circles drawn to make a T shape), and consist of the first three or four pattern terms. The position of each term is rarely displayed in the drawn representation. Students engaging with growing patterns in this way often identify the recursive pattern rule, as their attention is not attracted to the two variables or the co-variable relationship between the two variables. Recent studies with young non-Indigenous students have confirmed the benefits of explicitly representing the underlying structure of the pattern and using semiotic signs to draw attention to this structure when generalising (e.g., Warren & Cooper, 2008; Radford, 2006).

There have been few studies that have focused on the act of grasping a generalisation. Grasping a generality is to notice a commonality that holds across all terms (Cooper & Warren, 2011). Radford (2006) asserts that the act of grasping a generalisation rests on perception and interpretation. This is an active process, and is dependent on the use of signs (gesture, speech, concrete objects) that indicate where the perceived object is located. Radford's study (2006) focused on better understanding the role of signs in students' perceptive processes underpinning generalisation of number and geometric patterns.

While the theory of semiotics has been long established, it is only recently that studies in the area of pattern generalisation have considered how semiotics impacts on the learning process (e.g., Radford, 2006; Miller, 2014; Warren & Cooper, 2008). For example, aspects of the various semiotic resources (gestures, language, materials) used by students and teachers when exploring mathematical generalisations (Radford, 2006; Miller, 2014; Warren & Cooper, 2008) have been delineated. The gap still remains in the research with respect to how semiotics assists young Indigenous students to attend to both variables in growing pattern representations. Hence, the theoretical construct underpinning this research was semiotics.

Theoretical Framework

Semiotic signs assist students in developing mathematical understanding (Sabena, Radford, & Bardini, 2005). Semiotics is the study of cultural sign processes, analogy, communication, and symbols. Signs (such as bodily movement, oral language, concrete objects) play the role of making the mathematics apparent (Radford, 2003). As the teaching of mathematics draws on a variety of representations and resources to assist students to engage with mathematical processes, semiotics provides the tools to understand these processes of thought, symbolisation, and communication. Semiotics has a two-fold role in this study. First, it informed the selection of materials used to represent the growing patterns, and second, it provided the lens to interpret the signs within and between all social interactions in the learning experiences. The semiotic terms from Saenz-Ludlow and Zellweger (2012) model, adapted from Peircean theory (Peirce, 1958), are adopted for this study. Figure 1 displays the triadic concept of sign that has been developed with the classifications of sign object, sign vehicle and sign interpretant (Saenz-Ludlow & Zellweger, 2012).
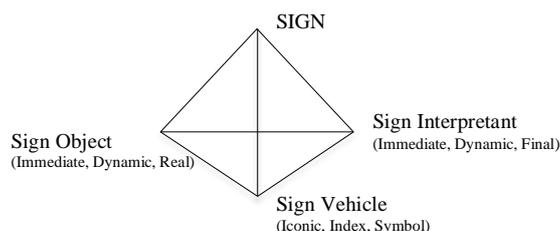
Figure 1. The tridactic concept of sign with terminology adapted by Saenz-Ludlow and Zellweger, 2012.

For the purpose of this paper, the focus is on sign vehicles. Sign vehicles are mediators between the sign objects and sign interpretant (student/teacher), and are then deduced to attempt to build understanding of the overall concept. The sign object can be represented by different sign vehicles that capture particular aspects of the object. Multiple sign vehicles are required, as one sign vehicle cannot encapsulate the entire object. To come to a complete understanding of the concept, students must be exposed to multiple and interrelated sign vehicles. These sign vehicles can be classified as iconic, indexical or symbolic (Saenz-Ludlow & Zellweger, 2012). Iconic signs exhibit a similarity to the subject of discourse (object); the indexical sign, like the pronoun in language, forces the attention to the particular object without describing it; and the symbolic sign signifies the object by means of an association of ideas or habitual connection (Peirce, 1958). These sign vehicles can be both static and dynamic (Radford, 2006; Saenz-Ludlow, 2007).

Research Design

A decolonised approach has been adopted with a focus on valuing, reclaiming, and having a foreground for Indigenous voices (Denzin & Lincoln, 2008). For this particular study, a relationship needed to be cultivated with Indigenous Education Officers (IEO) to assist with knowledge that may not be explicitly recognisable to the researcher. It was thus imperative to create space for critical collaborative dialogue within the study; hence the choice of teaching experiments to collect data. This is because teaching experiments provided the platform to investigate the teaching and learning interactions that support the development of students' ability to generalise in students' own setting. In effect, this brings the researcher and the participants into a shared space, where empowerment can occur (Denzin & Lincoln, 2008).

The larger project was based on two conjecture-driven teaching experiments for the primary purpose of directly experiencing students' mathematical learning and reasoning in relation to their construction of mathematical knowledge. A crucial aspect of the conjecture-driven teaching experiment is the conjecture itself. It needs to be aimed at both theoretical analysis and instructional innovations (Cobb, Confrey, DiSessa, Lehrer, & Schauble, 2003). Conjectures are based on inferences, and within mathematics education, these inferences may pertain to how mathematics is organised, conceptualised, or taught in order to reconceptualise the content and pedagogy (Confrey & Lachance, 2000). Each teaching experiment consisted of three 45-minute mathematics lessons, therefore six lessons in total. The researcher conducted the lessons in the study. Three conjectures were explored in each lesson, a mathematical (e.g., Exploring growing patterns where the structure is multiplicative (e.g., double) assists students to generate the pattern rule), semiotic (e.g., Providing growing patterns where the variables are embedded in the pattern ensures that students attend to both variables*),* and cultural conjecture (e.g., Exploring growing patterns from environmental contexts assists Indigenous students relate growing patterns to their prior experiences). Due to space limitations, this paper only draws from the first teaching experiment (teaching experiment 1), focusing on the semiotic conjectures from two lessons conducted with Year 2/3 Indigenous students.

Data Collection

The research was conducted in one Year 2/3 classroom (7-9 year olds) of an urban Indigenous school in North Queensland. This school was purposively selected because these students had not previously engaged in mathematics lessons focusing on the concept

of growing patterns. Additionally, a relationship was already formed with the school community, an important aspect of Indigenous research perspectives, and the researcher had worked in the classes as part of a larger mathematics research project. In total, 18 students and 2 Indigenous Education Officers (IEOs) participated in the study. To explore how students engaged with the growing patterns, data-gathering strategies used in teaching experiment 1 were: (a) A pretest to ascertain what the students knew before the lessons; (b) video-recorded mathematics lessons, and (c) audio-taped interviews with the IEOs. There were two video cameras in each lesson, one focused on the researcher and the other on the students. Data reported in this paper are from two students (S1 - Aboriginal girl & S6 – Aboriginal boy). These students were selected, as they represent cases of mathematical achievement, high (S1) and low (S6) achiever in mathematics, as identified by the classroom teacher and IEO.

Data Analysis

In this study, data analysis was contemporaneous and formative during data collection. It informed each stage of the data collection process and assisted in refining conjectures (Confrey & Lachance, 2000). Pretests were analysed not only to ascertain what students knew, but to also determine the ways in which the students answered the questions. This analysis informed selection of tasks for the first lesson of teaching experiment 1. The analysis of the videotaped lessons formed a major component of the qualitative data analysis. The teaching experiments required two phases of data analysis, ongoing analysis and in-depth analysis. Ongoing analysis occurred at the conclusion of each lesson of the teaching experiment, and informed the next stage of data collection. This assisted with refining conjectures and hypotheses, and the development of tasks for the next lesson (Confrey & Lachance, 2000). Peer debriefing between the researcher, supervisor, teacher, and Indigenous Education Officers was conducted at this point to determine conjectures for the following lesson. Member checks occurred during the teaching experiments with students to ensure that the researcher had correctly interpreted each student's response.

In depth analysis occurred at the conclusion of the data-collection phase. All data were reanalysed using an iterative approach (Srivastava & Hopwood, 2009). Initial video-footage were transcribed to capture students' verbal responses and the semiotic interactions. Data were coded and analysed focusing on semiotic signs (iconic and indexical) of both the student and researcher. This entailed identifying signs that assisted students to engage with the growing pattern structures. Finally, the data were reanalysed and aligned with the cultural perspective provided from the Indigenous Education Officers with regard to the semiotic signs. Their input was audio-recorded and then transcribed in order to capture cultural interactions in the lesson.

Findings

The data from S1 and S6 are presented by structuring the findings according to the order of data collection. First, results from the pretest that served to ascertain what students knew prior to the commencement of the lessons, and second, the conjectures that framed the three lessons in teaching experiment 1 are presented. *Pretest*: The test comprised 10 items. Figure 2 illustrates Student 1 and Student 6 responses to Question 3 (Copy the pattern) and 7 (How many possum eyes will there be if there were 10 possums hanging on the tree?) of Pretest 1, key questions that illustrate the differences in understanding between the two students, and the data that informed the development of Conjecture 1

(Making both variables of growing patterns visually explicit assists students to identify the co-variational relationship).
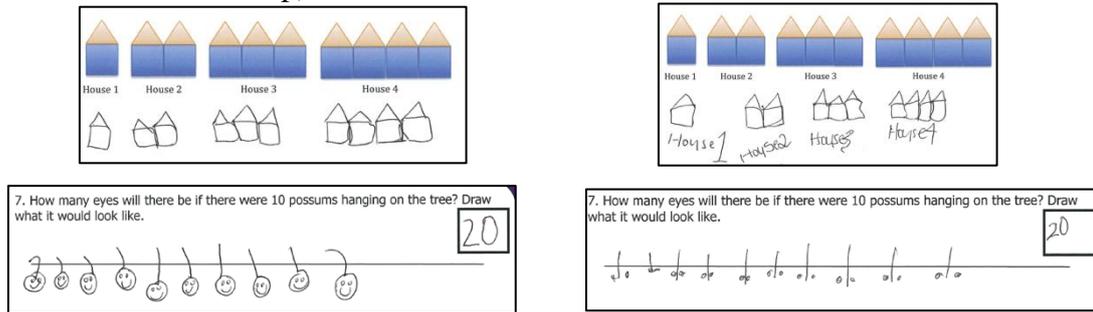

Figure 2. S1 and S6 responses to Question 3 and 7 of Pretest 1.

For Question 3 S1 only copied the houses in the pattern. She did not attend to both variables in the pattern (the houses (pattern term)) and corresponding label (pattern position). By contrast, S6 attended to both variables in the pattern. For Question 7 S1 and S6 started to work with both variables in the possum pattern. They both drew the possum tails and eyes. It appears that having the two variables (the possum tails and possum eyes) embedded in the single pattern structure (a drawing of a possum) assisted S1 and S6 to attend to both variables. At the conclusion of the Pretest, Indigenous Education Officers shared that within their context they could not identify any growing patterns that were appropriate for these young students to engage with. They suggested that it would be best for students to begin exploring patterns from a shared context (e.g., environmental context). They also confirmed that a hands-on approach (using concrete materials) would be appropriate for these students. Figure 3 presents the following patterns used in lesson 1 (butterfly pattern), and lesson 3 (kangaroo pattern).
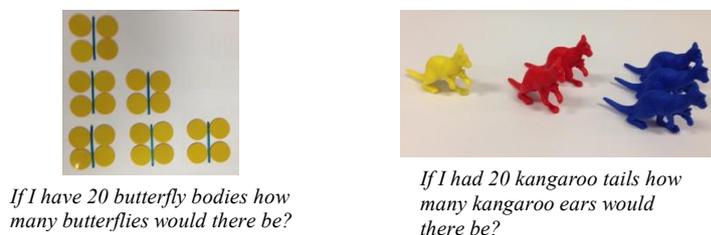
If I have 20 butterfly bodies how many butterflies would there be?

If I had 20 kangaroo tails how many kangaroo ears would there be?

Figure 3. Growing patterns used in teaching experiment 1.

Lesson 1 Conjecture: Making both variables of growing patterns visually explicit assists students to identify the co-variational relationship. During lesson 1, while both variables were visually explicit (blue matchstick – pattern term, yellow counter – pattern quantity) and embedded in the butterfly pattern, students attended to the iconic signs (matchsticks and counters – iconic signs) separately. When considering a butterfly in the natural environment the body and wings cannot be separated. However, when using the concrete materials, the sign vehicles were easily separated. S1 did not split the two signs; S1 placed one matchstick on her desk and then immediately placed the four counters around that matchstick before constructing the next butterfly. S1 was able to copy and continue the structure identical to that presented by the researcher (see Figure 3). It was for these reasons that S1 was considered to have high structural awareness of the butterfly pattern. S6 attended to the sign vehicles separately. Other students in the class also attended to the pattern in this manner. First, he placed an array of matchsticks on the desk

to represent the butterfly bodies, and then added the counters (wings) retrospectively. Thus, when constructing the pattern, S6 attended to the two sign vehicles (the iconic signs) sequentially rather than simultaneously. Whether he recognised the co-variational relationship between the two sign vehicles is difficult to determine. It is because of these actions, separating the sign vehicles, that the pattern for lesson 2 was selected (See Figure 2). Additionally, the IEO stated that the students were confident using the number ladder, and they were able to 'act out the pattern' by standing on the 'feet'.

Lesson 3 Conjecture: Providing growing patterns where only two variables are embedded and cannot be physically separated from each other, assists students to attend to both variables simultaneously. S1 was now attending to both variables when working with the kangaroo pattern. She was able to express further predictions of the pattern using both the tail and the ears to communicate her understanding. S1 explained to the class that if she had 1 million tails she needed to double the number of tails to determine the number of ears. She was also able to determine the number of tails if there were 10 kangaroo ears (five kangaroo tails). As both variables were embedded in the kangaroo pattern, and could not be separated, this assisted S1 to attend to both variables when discussing the pattern. S6 was able to attend to both variables in the kangaroo pattern to assist him explain the relationship between the tails and ears. He was able to predict how many ears there would be if there were 100 kangaroo tails (200), and explained that he was doubling the number of tails to find the number of ears.

Discussion and Conclusion

It has been demonstrated in past research that young non-Indigenous students can engage in covariational thinking (Blanton & Kaput, 2005); however, this current study adds new knowledge to the pattern task types that assist young Indigenous students in 'noticing' the relationship between two variables. Past research has highlighted an issue that arises from covariational thinking is the need to coordinate two data sets, and identify the relationship between these sets (Blanton & Kaput, 2005). Thus, in this present study the growing patterns selected for the tasks were deliberately chosen to ensure that this relationship was transparent. Results from this study provide initial evidence that iconic signs appear to assist students to move quickly from recursive thinking to covariational thinking. This was achieved by using iconic signs to highlight the two variables. Additionally, a recursive approach to solving growing patterns is still a major challenge for both young and older students (Rivera & Becker, 2009; Warren, 2005). The results of this present study suggest that this issue relates to the way the patterns are structured, and can be overcome by using iconic signs to highlight both variables in growing patterns, namely, the pattern number (term) and the pattern quantity.

While it is recognised that signs play a central role in the construction of new knowledge (Peirce, 1958; Saenz-Ludlow, 2007), literature pertaining to how these signs are represented in pattern generalisation tasks is scarce. This present study begins to contribute to this limited research, and suggests that there are two potential ways that sign vehicles can be considered when constructing growing patterns: (a) embedding sign vehicles (possum and kangaroo pattern), and (b) splitting sign vehicles (house and butterfly patterns). When considering growing patterns the sign vehicles represent the two variables within the pattern (i.e., pattern term and pattern quantity). Embedding both sign vehicles in a single hands-on artefact ensures that students attend to both variables of the growing pattern. This aligns with past research, indicating that students were successful generalising patterns where both iconic signs were embedded in the single structure

(Blanton & Kaput, 2005; Leung, Krauthausen & Rivera, 2012; Warren, 2005). Young Indigenous students were supported in making connections with co-variation, as demonstrated by S1 in TE1 when discussing the general rule for the kangaroo pattern. It appeared that the use of *embedded variable patterns* assisted students to attend to both variables: that is, students needed to discuss the pattern attending to the pattern position (tails) and the pattern quantity (ears).

It appears that iconic sign vehicles (e.g., concrete materials) provide opportunities for dynamic interactions between the student and the pattern. Findings from this study further nuance the importance of the role that dynamic signs play when students physically engage with geometric patterns to construct the general rule (Mason, 1996; Saenz-Ludlow, 2007). Through the use of iconic signs (butterfly bodies and butterfly wings), a geometric pattern created with concrete materials provides opportunities for young students to manipulate both variables, as they examine the pattern structure on their way to constructing generalisations (Cooper & Warren, 2008). This approach differs from geometric patterns that are traditionally depicted in textbooks (as students can not physically manipulate textbook pictures), and it is argued that potentially students may not engage with, or interpret these signs (textbook pictures), with the same intensity. It is conjectured that growing pattern task should be set up to have, dynamic iconic signs that represent both variables, so that students can physically engage with these signs.

The contribution of this study is that growing pattern task design should mirror and support students use of semiotics as a thinking tool and as such one needs to consider signs in the representation of patterns. These tasks types have implications for both the teaching and learning of growing pattern generalisations. As two cases were presented, it is acknowledged that there are limitations for the study. Thus further research is needed to consider larger cohorts of both Indigenous and non-Indigenous students, to determine if these pattern tasks assist young students to engage in growing pattern generalisations, and if there is a potential hierarchy to which growing patterns should be introduced to young students. Finally, and most importantly, this study provides a positive story for Indigenous students indicating that they are capable of engaging with early algebraic thinking challenging deficit models of mathematics learning.

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