The latest curriculum development effort of the Math in a Cultural Context, a long-term Alaskan project, includes Indigenous knowledge (IK). Collaborating with Yup’ik elders, MCC has identified a powerful set of mathematical processes used in constructing everyday artefacts. The knowledge of elders provides a unique way to teach Rational Number Reasoning. Measuring the efficacy of curriculum developed from IK requires a reliable and valid assessment instrument, which captures the mathematical content and learning trajectory established by Indigenous knowledge. An appropriate assessment instrument was unavailable; hence adapting questions from other instruments was undertaken. This paper describes the process of adapting an Australian fraction assessment for use in this Alaskan context.

Context

The underperformance of schools serving American Indian (AI) and Alaskan Native (AN) students and communities has been one of the most vexing and enduring issues in education. Federal reports have, for almost a century, advocated approaches that recommend educational programs connect school and community (Meriam, Brown, Cloud, & Dale, 1928; Executive Order No. 13,336, 2004), as a way to redress the continuing lower academic performance of AI/AN students, particularly in the mathematics domain. To address this problem, the Math in a Cultural Context (MCC) project has developed a long-term curriculum and professional development project in collaboration with Yup’ik Eskimo elders from southwest Alaska. The project has expanded to both urban and rural school districts and has been implemented across Alaska’s diverse geographical and cultural regions: Athabaskan, Inupiaq, Tlingit and Yup’ik. The current project takes place within five diverse Alaskan school districts.

MCC curriculum development is underpinned by the use of everyday Indigenous activities that are mathematically rich, with the potential to engage students and improve their understanding. Subsistence activities such as gathering berries and constructing a fish rack have become the foundation for a supplementary elementary curriculum and associated professional development materials. Positive impacts on AN students’ mathematics performance have been observed when using MCC’s supplemental curriculum (Lipka & Adams, 2004; Lipka, Yanez, Andrew-Ihrke, & Adam, 2009; Sternberg, Lipka, Newman, Wildfeuer, & Grigorenko, 2006).
Repeatedly, elders have demonstrated how they use body proportional measuring and symmetry/splitting in tailoring clothing, constructing buildings, and star navigating. MCC’s approach is to work with Yup’ik Eskimo elders and Yup’ik teachers, mathematicians and math educators, educators and Alaskan school districts with an aim of integrating Yup’ik and Western knowledge for the purpose of improving students’ mathematics knowledge and performance (Lipka, et al., 2009). As this two-decade-old project has matured, we have increasingly recognised the mathematically laden ways that Yup’ik elders use their knowledge to solve everyday problems. Our most recent mathematics curriculum development and learning trajectory begins from Indigenous knowledge (IK) and the Indigenous worldview. Constituting mathematics curriculum from IK, that is both an authentic representation of Yup’ik cultural practice and school mathematics is a turnaround from the not so distant colonial past (Lipka & Andrew-Ihrke, 2009).

MCC is currently developing Rational Number Reasoning (RNR) and geometry curriculum materials that intertwine Yup’ik constructions with fractions, ratios, and proportional reasoning. Historically, elders did not and could not rely on exogenous tools to construct items so they employed body symmetry and body-part relationships as a precise form for measuring proportionally so their end-products (e.g., clothing, boots and kayaks) fit the user. Central to both Yup’ik everyday practices and the development of a RNR and geometry curriculum for elementary school students lies a set of generative concepts gleaned from elders’ practice. The dynamic way in which body proportional measuring and symmetry interact presents an integrated perspective on teaching measuring, geometry, patterns, numbers, and early algebraic thinking.

An important and common Yup’ik measure is the “knuckle”, which forms the basis for constructing a square, which can be transformed into geometrically pleasing patterns that adorn squirrel parkas or become the basis of circles used for ceremonial headdresses, as shown in Figure 1. In both cases, the knuckle measure is ½ the length of the constructed square and ½ the length of the diameter of a circle, thus establishing a 2:1 or 1:2 relationship. The square then becomes the base from which a circle is made—both are shown in Figure 2. Other Yup’ik body proportional measures are also used for constructing a variety of projects. For example, a kayak measure is approximately 3:1-Yagneq (arm span) to the length of a kayak.

(a) Pattern on a squirrel parka
(b) Ceremonial headdress

Figure 1. Yup’ik artefacts.
Like existing MCC modules, the RNR curriculum incorporates tasks to engage students in creating their own representational models. Using the knuckle measure, students’ first construction is creating a square from uneven material. Through the process, students and teachers observe how symmetrical splits create congruent areas (see Figure 3), learn basic Euclidean geometry—2 points create a line, parallel and perpendicular lines. Rather than verifying the square using an Euclidean proof that the four sides are equal length and all angles are right angles; from a Yup’ik perspective, the square is verified using transformational geometry, “It is about what you do to the shape that stays the same … that is a reflection … the two sides of the mirror—the image and the original match” (Lipka & Andrew-Ihrke, 2009, p. 9). Students learn that one-fold creates \( \frac{1}{2} \), a second or recursive fold creates \( \frac{1}{4} \), and a third-fold creates \( \frac{1}{8} \), which forms a foundation for multiplicative thinking.

Assessment of efficacy

The RNR project will adopt a similar research design to previous MCC curriculum development projects. Those projects employed a quasi-experimental pre/post-test design, in which intact classes were assigned as a control group or an experimental group (Lipka & Adams, 2004; Lipka, et al., 2009). All students were tested prior to the commencement of the teaching of the unit and at its completion. The experimental group were taught using MCC’s supplemental curriculum, while the control group used their usual curriculum materials, typically the district adopted mathematics text.

On previous occasions MCC have used or adapted assessment instruments available from other research projects. When no suitable instrument met their needs, pre-test and post-test instruments were constructed by selecting appropriate questions from the National Assessment of Educational Progress (NAEP) and *Trends in Mathematics: Traditions and [NEW] Practices*.
International Mathematics and Science Study (TIMSS). Items were also created to reflect the major mathematical components of the module. Instruments were also piloted to assess and compare the difficulty between pre-test and post-test instruments, and determine their reliability (Lipka & Adams, 2004).

Assessment of fraction understanding instrument

The Assessment of Fraction Understanding (Wong, 2009) was identified by the MCC team as an instrument that could be used or adapted for the RNR project. The instrument was intended for use to establish students’ level of knowledge and understanding of fraction equivalence. A learning pathway developed from empirical evidence enabled the three aspects of learning to be identified for students: (a) knowledge that has been mastered; (b) likely misconceptions that will be exhibited; and (c) knowledge required to further conceptual understanding (Wong, 2009, 2010).

The Assessment of Fraction Understanding (AFU) comprised two forms, one for grades 3 and 4, and another for grades 5 and 6. Form A comprised 25 constructed-response items, while Form B comprised 25 constructed-response items. Eighteen items were common across the two forms, which enabled students to be compared across grades without the need for all students to be administered all items (Wright & Stone, 1979). Items incorporated area models (i.e., circular, rectangular and square), number-line models, unit recognition, partitioning, equivalence and fraction language. A full description of the instrument, its development, and its testing is found in Wong (2009).

Assessment Adaptation

Modification of the AFU to meet MCC specifications required the addition of ratio and proportional reasoning items, and parallel forms for grades 2 to 6. MCC’s long-standing partnership with Yup’ik elders and teachers and the development of assessment instruments in previous projects, provided a process from which instrument development/modification was undertaken. The process used to adapt the AFU comprised six main steps:

1. Yup’ik elders demonstrate the cultural activity to be incorporated in the RNR curriculum, aligning the instrument to indigenous knowledge.
2. Explore the mathematics embodied within the cultural activity.
3. Present research on student learning of fractions, proportional reasoning and ratios.
4. Develop an item bank.
5. Develop/modify assessment instrument.

The first four steps of the process were undertaken at a weekend Teacher Leadership Workshop conducted by MCC, with Yup’ik elders, teachers and educators. Following introductions, Dora Andrew-Ihrke, a long-term MCC adjunct faculty and Yup’ik cultural expert, described and demonstrated, in English, cultural activities considered suitable for the RNR curriculum. Evelyn Yanez, also a long-term Yup’ik consultant to MCC, interjected occasionally with relevant Yup’ik words, explanations, and how Yup’ik stories can support RNR. They described how they visualise the process of
tailoring, creating patterns, and showed appropriate cultural artefacts. Both Dora and Evelyn responded to questions generated from their demonstrations. They then guided the workshop group in completing a number of activities, which enabled the participants to become familiar with mathematically embedded processes of body proportional measuring and splitting/symmetry.

The second step of the process was to identify and clarify the mathematics embodied in the cultural activity and re-contextualise the knowledge of elders to fit modern schooling (Lipka & Andrew-Ihrke, 2009). Discussion of the mathematics, such as constructing a square, how it could be incorporated in the classroom was undertaken. Teachers also explored how they could use the approach to develop fraction sets based on body proportional measuring and symmetry/splitting.

The next stage of the process was to present to the attendees, the learning trajectory or pathway identified by Confrey, Maloney, Nguyen, Mojica, and Myers (2009) for developing rational number understanding, and the pathway of learning linked to the AFU (Wong, 2009; Wong, 2010). Also, discussed were how learning pathways can inform teaching and learning fractions, ratios and proportional reasoning, and how the pathway would be recalibrated for indigenous knowledge.

Prior to creating items suitable for inclusion in the MCC assessment, a discussion of assessment design considerations was conducted; bias, common errors, types of problems (e.g., symbolic, pictorial, routine/non-routine, procedural, conceptual), item difficulty, clarity of instruction, and duration of assessment were discussed. Teachers and educators then worked in grade level groups to examine the applicability of AFU assessment items from the item bank and create items suitable for their grade, to assess the mathematical thinking embedded within the cultural activities demonstrated. Items created were catalogued and added to the item bank.

After the weekend workshop, pencil and paper assessments for grade 2, grade 3–4 and grade 5–6 were created and emailed to the teachers for review. From the comments received, the assessments were revised and two versions, A and B created. Both versions comprised the same number of items, however for three grade 2 items, five grade 3–4 items and one grade 5–6 item, however one version incorporated a diagram that was absent from the other. For example, item 6 from grade 3–4 version B is shown in Figure 4; the diagram was omitted in the version A.

<table>
<thead>
<tr>
<th>Grade 3-4 (version B)</th>
<th>Grade 5-6 (version A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6. For every 2 King Salmon there are 3 Reds.</td>
<td>9. Mark and John have identical candy bars. Mark ate ( \frac{4}{7} ) his candy bar and John ate ( \frac{2}{7} ) his candy bar. Who ate more?</td>
</tr>
<tr>
<td>If a fish rack holds 6 King Salmon, how many Red Salmon would the fish rack hold?</td>
<td>Grade 5-6 (version B)</td>
</tr>
<tr>
<td>9. Mark and John have identical candy bars. Mark ate ( \frac{1}{7} ) his candy bar and John ate ( \frac{1}{5} ) his candy bar. Who ate more?</td>
<td></td>
</tr>
</tbody>
</table>

Figure 4. Sample items from the grade 3-4 and grade 5-6 assessments.

For the grade 5–6 assessment, the fraction quantity in four items varied across versions. An example is shown in Figure 4. Common items, such as the king salmon item, were used to link the instruments across grades and versions. Items were also
retained from the AFU unaltered to enable comparison of learning pathways from different samples.

Pilot testing of the MCC assessments

The development of the assessment and its piloting was presented to teachers as a recursive process which continued until the assessments were tested and verified statistically to be both reliable and valid. Therefore, teachers and educators who attended the weekend workshop agreed to pilot the new assessments at their schools. Three iterations of piloting and modification of the instrument were undertaken and discussed as follows.

First Iteration: Versions A and B

The first round of pilot testing was conducted by a MCC staff member and the first author at the Alaska Native Cultural School, a school with a majority of AN students, in Anchorage, Alaska’s largest city. The assessments were administered grade 2 to 6 students, with the number of participants and the version they completed shown in Table 1.

Table 1. Sample of students tested by version and grade.

<table>
<thead>
<tr>
<th>Grade</th>
<th>Version A &amp; B</th>
<th>Version C</th>
<th>Version D</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>18</td>
<td>26</td>
<td>92</td>
</tr>
<tr>
<td>3</td>
<td>20</td>
<td>20</td>
<td>89</td>
</tr>
<tr>
<td>4</td>
<td>17</td>
<td>12</td>
<td>70</td>
</tr>
<tr>
<td>5</td>
<td>24</td>
<td>16</td>
<td>99</td>
</tr>
<tr>
<td>6</td>
<td>23</td>
<td>18 (version A)</td>
<td>122</td>
</tr>
</tbody>
</table>

During test administration, students were asked to work alone; they could ask for clarification of questions and were offered paper for folding. Grade 2 students were administered each assessment item verbally, with the administrator reading each question aloud to the whole class. An overhead projector was also used to guide students through the assessment and to ensure answers were written in the correct location. The time taken for students to complete the assessment, their composure and actions were observed during assessment administration for all grades.

It was observed by both test administrators that students in grades 2, 3, 4, and 5 had difficulties completing their assessment. Hence, marking the completed assessments at the point of data collection was undertaken. Rather than determine a total score for each completed assessment, responses for individual questions were examined, along with a comparison of responses for items with pictures and no-picture, and grade 5–6 items with different fraction quantities. This type of review also provided an indication of item difficulty (Bond & Fox, 2001).

Responses to the king salmon question are listed in Table 2, stratified by picture/no-picture, grade and response. The correct answer 9 appears in bold type. The number of responses for answers 3, 6, and 18 are included, along with an “other” category, which includes whole number answers not listed, answers with fractions, and “non-attempts”. Of the grade 5 students who completed the picture question, 27%
(n = 26) answered it correctly, compared to 50% (n = 8) who completed the non-picture version. Of the grade 6 students who completed the picture question, 36% (n = 28) answered it correctly, compared to 60% (n = 5) who completed the non-picture version. Determining whether pictures provided an advantage was not possible due to the small sample sizes. Review of all items within all assessments confirmed the assessments were too difficult for grades 2 to 5.

Table 2. Responses to the King Salmon Question

<table>
<thead>
<tr>
<th>Picture</th>
<th>Responses</th>
<th>No-picture</th>
<th>Responses</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grade 2</td>
<td>3 6 9 18</td>
<td>other</td>
<td>Grade 2</td>
</tr>
<tr>
<td>2 (n = 9)</td>
<td>2 1 0 1 4</td>
<td>2 (n = 10)</td>
<td>0 5 1 0 4</td>
</tr>
<tr>
<td>Grade 3</td>
<td>0 0 1 1 6</td>
<td>3 (n = 12)</td>
<td>0 1 2 1 8</td>
</tr>
<tr>
<td>3 (n = 8)</td>
<td>0 0 3 0 3</td>
<td>4 (n = 11)</td>
<td>0 1 3 1 6</td>
</tr>
<tr>
<td>Grade 4</td>
<td>1 1 7 1 6</td>
<td>5 (n = 8)</td>
<td>1 1 4 1 1</td>
</tr>
<tr>
<td>4 (n = 6)</td>
<td>1 1 7 1 6</td>
<td>5 (n = 8)</td>
<td>1 1 4 1 1</td>
</tr>
<tr>
<td>Grade 5</td>
<td>0 0 10 3 5</td>
<td>5 (n = 5)</td>
<td>0 1 3 1 0</td>
</tr>
</tbody>
</table>

Discussions of the difficulty of the assessments were undertaken with the classroom teachers and MCC principal investigator, and it was decided that adjustments to the assessments were needed prior to visiting the second school the following day. From the results of the review and observations during assessment administration, no adjustments were made for grade 6. Major revisions as listed, were undertaken resulting in the creation of version C:

- Grade 2 – Reduce the number of items and incorporate items with diagrams.
- Grade 3-4 – Use the grade 2 versions as the basis for creating a new instrument and add some difficult items.
- Grade 5 – Use the grade 3-4 versions for creating a grade 5 instrument.
- Ensure adequate link items across all grades.

Some items were also reworded or reorganised. For example, the fish in the king salmon item (see Figure 4) were repositioned vertically as they would appear in real life. An item aimed at addressing the paper folding process was also reviewed for clarity of instruction, as the pictorial representation of the process was ambiguous. With the assistance of classroom teachers, a number of attempts at rewording the item highlighted the difficulty in creating pencil and paper items which reflect the underlying mathematical concepts revealed by Dora’s cultural activities. Hence a companion performance-based, hands-on assessment (one-to-one interview) was created for administration to a subset of students who also completed the pencil and paper assessment.

2nd Iteration: Version C

The second round of testing was conducted at Dillingham City School, a rural school with a majority of AN students, within the Bristol Bay region. Version C was administered to students from grades 2 to 5, while versions A and B of the grade 5-6 assessment, were administered to grade 6. The number of students tested is shown in
Table 1. The process of reviewing the response to individual items as undertaken in the first iteration was also undertaken for all grades, which confirmed the assessments comprised items with a range of difficulties. No items had either a 100% or zero percent success rate.

One-to-one interviews with the first author were conducted with four of the students: (a) to ensure questions were interpreted as intended; (b) to gauge the difficulty of the items and assessment overall, (c) to identify the mathematical thinking exposed by the question; and (d) to uncover likely strategies to be employed. Those students did not undertake the assessment with their class. It was found that item wording did not pose a problem to answering the items and different strategies were employed by students to answer items.

Iteration 3: Version D

Final changes to the assessments were undertaken to ensure consistent representation of items across and within forms. For example some fractions were in-text (e.g., 1/4, 5/8), while others were in vertical format (e.g., \( \frac{1}{4} \)); all were changed to the vertical format. Both authors administered Version D at five elementary schools and one middle school in Juneau, the capital of Alaska. Not all grades or classes within grades were tested at each school.

The analysis of grade 3-4 data was undertaken first as this instrument contained the greatest number of common items between the grade 2, grade 5 and grade 6 instruments. The assessment was shown to be reliable using Cronbach’s alpha = .88, \( n = 159 \), calculated using SPSS v16. Using RUMM2020, all but three of the items in assessment fit the dichotomous Rasch model (RUMM Laboratory, 2004a). These items will be reviewed to determine any necessary changes.

Data coding and preliminary analysis for grade 6 assessments is underway. Although Cronbach’s alpha = .90, \( n = 122 \), review of the instrument and responses to items showed that further rewording of items is necessary. Two items were found to violate the assumption of local independence during Rasch modelling, hence were omitted from a second analysis. Further results showed that four items violated the assumptions of item fit and three polytomous items exhibited disordered thresholds (RUMM Laboratory, 2004b); these items require further analysis and review with changes to the grade 6 assessment expected.

Once analysis of all grade level assessments is complete, the data will be aggregated and Rasch modelling conducted on the entire data set. This will provide a preliminary learning trajectory commensurate with learning fractions, ratios, and proportional reasoning from indigenous knowledge.

Conclusion

The RNR pencil and paper assessments were designed to reflect IK knowledge. To do so, it was imperative that the assessment developers and teachers understood the cultural activities and mathematics embedded within those activities. One difficulty encountered in developing a culturally valid instrument was preserving the cultural knowledge in an authentic form. Three iterations of development and testing were undertaken. Preliminary analysis shows that further item development is needed to improve instrument reliability and validity. The adaptation of the AFU instrument to
another cultural context presented enormous challenges. However, the possibility of establishing an assessment instrument that reflects IK and calibrates a learning trajectory that follows the cultural activities and learning process gleaned from Yup’ik elders’ knowledge represents “a first.” The refinement process is expected to continue during the next few years.

Acknowledgement

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