24-11-2018

Innovative mathematics curriculum and teacher professional learning: A case study

Laurinda Lomas

Innovative Mathematics Curriculum and Teacher Professional Learning: A Case Study

Laurinda Lomas

BA (University of Adelaide) DipEd (University of Sydney)

MEd (University of Technology Sydney)

Faculty of Education and Arts

Submitted in fulfilment of the requirements for the degree of Doctor of Philosophy

December 2018

Statement of Authorship and

Sources

This thesis contains no material that has been extracted in whole or in part from a thesis

that I have submitted towards the award of any other degree or diploma in any other tertiary

institution.

No other person's work has been used without due acknowledgement in the main text of

the thesis.

All research procedures reported in the thesis received the approval of the relevant

Ethics/Safety Committees (where required).

Signature:

Laurinda Lomas

Date: 24 November 2018

Statement of Appreciation

My acknowledgments of appreciation are both broad and specific. All are genuinely felt.

Thank you to the Australian Mathematics Education research community for their support and encouragement; it is a privilege to work beside giants in their field.

Thank you to my dear friends and teaching colleagues who always believed I could do this (even when I was unsure) and made me laugh at all the right times.

Thank you to my supervisor Assoc. Prof. Janeen Lamb whose expertise and positivity was always invaluable.

Thank you to my supervisor Prof. Doug Clarke who has been my inspiration for much longer than the period of this journey and will be so for as long as I can think of.

And most of all, thank you to all my family: to my mum Elizabeth who always taught me to be curious and resourceful and to my dad Ronald who celebrated the search for knowledge and ideas, to my sisters Michelle and Simone and their families who always offered quiet encouragement, and to my parents-in-law Harry and Elsie who have supported me in so many ways for so long.

To my husband Graeme and our amazing children Jack, Emma, and Harry, I offer my greatest thanks and devotion. Love like yours is what drives me most.

I acknowledge professional editor, Pam Firth, who provided copyediting and proofreading services, according to the guidelines laid out in the university-endorsed national 'Guidelines for editing research theses'.

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Abstract

Implementation of innovative curriculum is posited as an immersion strategy with the potential to deepen the mathematical knowledge of in-service teachers. The impact of beliefs on teacher practice, however, can be a substantial constraint in this change process. The purpose of this research was to explore the impact of teaching an innovative mathematics unit on teachers' knowledge and beliefs about mathematics and mathematics teaching and learning.

Using a case study methodology within a constructivist epistemology, two Year 5/6 teachers were studied as they taught a rational number unit of work originally developed in the Netherlands, underpinned by Realistic Mathematics Education (see, e.g., Streefland, 1991).

The interconnected model of teacher growth (Clarke & Hollingsworth, 2002) provided the theoretical model of change in this study, and the mathematical knowledge for teaching model (Ball, Thames, & Phelps, 2008) was used to analyse knowledge growth.

The initial relationship between the case study teachers and innovative curriculum was pivotal in determining curriculum fidelity and the potential for meaningful change. Both teachers were very experienced and had volunteered for the immersion experience, but issues of trust and teacher authority constrained long-term change. Neither of the teachers in this trial

took the full opportunity to learn afforded by the innovative curriculum because they did not seem prepared to reflect seriously on their established beliefs about teaching in general.

One of the teachers was open to learning new ways to teach fractions and was pleased at the surprising responses of students he had previously considered lower achieving. This resulted in subject matter and pedagogical content knowledge growth, as he considered the problem-solving strategies promoted by the unit better than those he had used in his career.

The second case study teacher's negative attitude towards the innovative curriculum based on previous experiences overseas resulted in constant concern about extending higher achieving students. She found managing the dissonance of her goals for teaching with those of the innovative curriculum extremely difficult to reconcile, resulting in a reduced number of lessons taught, little long-term growth in knowledge, and no apparent change in beliefs.

This research suggests that innovative curricula like those based on Realistic Mathematics Education have the *potential* to challenge the conceptual schema of teachers, but only if they are open to such experiences. Considering the resilient influence of beliefs on teacher practice

observed in this research and its status in the change environment, strategies that both challenge

and support teachers are needed if serious and sustained growth is to be realised.

List of Abbreviations

AITSL Australian Institute for Teaching and School Leadership

CAMCC cognitive-affective model of conceptual change
CBAM concerns-based adoption model

CCK common content knowledge DoE
Department of Education

DC domain of consequence DP
domain of practice

ED external domain ERIC Education Resources
Information Center

FI Freudenthal Institute GTE general
teaching efficacy

HCK horizon content knowledge IMTPG interconnected model
of teacher professional growth

KCC knowledge of content and curriculum KCS
knowledge of content and students

KCT knowledge of content and teaching LCM
lowest common multiples

LST learning and support teacher MCK
mathematical content knowledge

MiC Mathematics in Context MKT mathematical
knowledge for teaching

MSP Mathematics and Science Partnership NNF
National Numeracy Fund

NSW New South Wales PBEL positive behaviour
to engage learning

PD personal domain PCK pedagogical
content knowledge

PoP patterns of participation

PTE personal teaching efficacy PUFM profound understanding of fundamental mathematics

RME Realistic Mathematics Education SCK specialised content knowledge

SMI school mathematics images SMK subject matter knowledge

SoC stage of concern

TALIS Teaching and Learning International Survey TBS Teacher Beliefs Study

TE teacher efficacy TIMSS The International Mathematics and Science Study

TPL teacher professional learning TTML Task Types and Mathematics Learning

UK United Kingdom US United States

Chapter 1: Introduction

1.1 CHAPTER OVERVIEW

The purpose of this chapter is to outline the research problem that framed my study. The chapter addresses the need for innovative approaches to professional learning that support in-service teachers to meet the increasing demands of current mathematics syllabus documents. Central to the argument is the tension between teacher knowledge and beliefs about mathematics and the innovative approaches to teaching mathematics espoused in the research literature, and the capacity of schools to address these. In the current Australian educational environment, schools are under increased pressure to develop the problem-solving skills of students yet less able to access expert support to assist teachers with their own conceptual understanding. This chapter acquaints the reader with my own experiences of such constraints in various professional roles. It provides related background to generally held concerns about Australian teachers' mathematical knowledge and student performance at national and international levels. I also draw on the expectation of continued professional learning to maintain a teaching body with the capacity to address 21st century skills, along with the response from Australian and international governing bodies in this endeavour. An international approach to mathematics curriculum design and teacher support materials is introduced in the

preamble to articulation of the research problem and the research question.

1.2 PERSONAL CONTEXT

I am currently a deputy principal (non-teaching) at a Northern Beaches public school in

Sydney, New South Wales (NSW), a position I have held for five years. Our school is situated

in an upper-middle-class area of the Northern Beaches of Sydney. There are 23 classes from

Kindergarten to Year 6, with a total student population of around 580. The teaching staff

comprises 26 full-time and part-time teachers, including a teacher–librarian. The executive

structure includes the principal, deputy principal, and four assistant principals.

For the last 23 years, I have held a variety of positions strongly connected by my desire

to teach and support both students and teachers. These roles include classroom teacher, gifted

and talented specialist, statewide mathematics consultant, and on-class school executive in both

the public and independent education sectors. In my role as deputy principal, I have both

administrative and professional development responsibilities. I supervise teachers through

accreditation processes, observing their practice and providing relevant feedback to improve

practice.

In my two-year position as mathematics consultant for the Association of Independent Schools of NSW, my role included attempting to model research-based practice through team teaching, mentoring of teachers through lesson observation and reflection, and leading professional development sessions. My position as consultant coincided with the introduction of the then recently developed NSW Mathematics Syllabus (Board of Studies, 2002). In its philosophy and model of learning, this syllabus presented mathematical process as central to the learning of mathematics content. The process strand called Working Mathematically made clear to teachers that mathematics content must be addressed through tasks that provided students the opportunity to ask questions, apply strategies, communicate, reason, and reflect. In this direction to teachers, the syllabus clearly incorporated respect for student thinking and different approaches to problem-solving.

As a consultant and then school executive member, I had a range of organisational, leadership, and curricula responsibilities within my own school and across our community of schools. As a consultant, I was immersed in a variety of primary school settings across public and private sectors, observing and supporting teacher practice. I noticed a lack of deep knowledge of mathematics and teaching mathematics. Many teachers did not seem to have a conceptual understanding of the mathematics they were teaching. They were unable to break down complex ideas and/or see the key ideas underpinning the facts, skills, and procedures they taught their students. When I discussed this with my teachers, it challenged their self-concept and role as teacher. This personal challenge presented in a variety of forms. Some teachers were anxious about their own deep understanding; they felt unable to plan lessons that matched curriculum expectations owing to their surface knowledge of the content involved. Others were confident that rote learning and teaching the basics was the most effective pedagogical approach and as a result were resistant to any suggestion otherwise. This observation was not restricted to the career stage; it included early career to highly experienced teachers and even school leadership colleagues.

In dealing with this situation, I promoted teaching strategies that addressed content through the process outcomes outlined in the new mathematics syllabus. I assisted teachers in implementing assessment approaches that informed them about the different strategies students used to solve problems, thereby informing their instructional planning. This support was augmented with research-based professional learning sessions to deepen teachers' knowledge of the conceptual underpinnings of syllabus content. To reinforce these newly acquired

knowledge and skills, supportive observation of and reflection upon teachers' practice were offered. Student responses to tasks and planning for future lessons were also discussed. In my experience, such support requires trust from individual teachers. If teachers feel the support offered is loaded with judgment about their practice, they are less likely to immerse themselves in the experience.

As deputy principal, I am responsible for the leadership of teaching and learning within the school, including program supervision of teachers, assistant principals, learning and support teachers, and teachers' aides. I oversee beginning teacher accreditation and mentoring, including direct observation and support of classroom practice. The age and experience of teaching staff at the beginning of this research was highly variable; eight teachers had less than five years' experience and 10 had more than 30 years' experience. More than half the staff were under 35, whereas all but two of the remainder were over 55. This age and experience disparity continues to be challenging as the professional learning needs of such a diverse staff are so different.

An important part of my role is to support teachers to implement all Australian syllabi, taking into consideration the effect of imperatives set by state and national bodies and necessary teacher knowledge and skills to meet such expectations. Anecdotal evidence from my observations of programs, tasks, work samples, and teaching methods at my current school indicated an emphasis and value on procedural fluency and memorisation of number facts.

Algorithmic (step-by-step) procedures were introduced early in Stage 2 (Years 3 and 4), where worksheets (booklets) formed the basis of most lessons. Mathematics textbooks underpinned lesson structure up until access to class sets was removed by the newly appointed principal who wanted to promote differentiation, remove reliance on one source of teaching materials, and encourage teachers to make greater reference to the syllabus itself. This situation reinforced my previous experience, making me increasingly aware of how much support in-service teachers required to meet the demands of the syllabus and how challenging this support process could be for both those offering support and those receiving it.

As deputy principal, I also analyse school performance data to design school targets, including those related to teacher professional learning. Examining results from the National Assessment Program—Literacy and Numeracy (NAPLAN) can provide a snapshot into the depth of student understanding of selected mathematics topics. Such analysis can reveal patterns in student responses that suggest the presence of a procedural approach to teaching in substrands that require conceptual understanding as they become more advanced and complex.

Administrators of the NAPLAN assessment (Australian Curriculum, Assessment and Reporting

Authority; ACARA) provide schools with an indication of expected growth for students at each achievement level (band) in the two years between each assessment.

At the beginning of this study in 2012, my school's growth statistics in NAPLAN from Year 3 to Year 5 showed nearly 79% of students achieved greater than or equal to expected growth in numeracy.

In school-based evaluation of these assessment results, notice is taken about the percentage of students who achieved the expected growth from Year 3 to 5 and from Year 5 to 7. In 2012, my school's growth statistics from Year 3 to 5 met the school's locally set minimum growth targets, suggesting that many of the strategies used to promote growth up until Year 5 were successful. The growth results from Year 5 to 7, however, showed cause for some concern. Only 34% of students achieved greater than or equal expected growth in numeracy. As concepts build on earlier understanding and become more complex in Stage 3 (Years 5 and 6), a solid basis to link ideas is needed for understanding, and reliance on isolated procedural knowledge becomes problematic. Individual question analysis for students in Year 5 in 2012 showed a considerable increase in-group difference from similar schools. Fractions and Decimals and Patterns and Algebra, substrands that require deep conceptual knowledge bases as difficulty increases, stood out, being eight out of 10 of the worst performed items. On two of these stage-appropriate (Stage 3) items, students from our school performed below the state (NSW) average.

In combination, these results flagged a need to investigate the teaching and learning of mathematics at the school, particularly the understanding of students in mathematically complex concepts like fractions and decimals and patterns and algebra. They also warranted enquiry into, and possibly development of, teachers' pedagogical content knowledge in relation to these complex topics.

1.3 NATIONAL AND INTERNATIONAL CONTEXT

My concerns about results in my school context were also reflected in the national discourse around student performance. International comparative performance assessments form part of the Australian Government's National Assessment Program. Australia has participated in the Trends in the International Mathematics and Science Study (TIMSS) every four years since 1995. The assessment has a curriculum focus, reflecting the extent to which Australian students in Year 4 and Year 8 have gained knowledge and understanding of the curriculum content domains of Number, Geometric Shapes, Measurement, and Data. Australia

has generally performed well in the past on such international measures, although there is acknowledgement of slippage in recent years.

Despite having previously performed at a similar level in 2007, 2011, and 2015, Australian Year 4 students were outperformed by students in all participating Asian countries and by the United Kingdom (UK) and United States (US) (Thomson, Wernert, O'Grady, & Rodrigues, 2017). Lack of progress in Australian Year 4 and Year 8 student scores is also evident. For the past three TIMSS cycles, Australia's Year 4 overall mathematics score has remained unchanged, and the Year 8 mathematics score in 2015 was very close to that recorded 20 years earlier (Thomson et al., 2017).

In their analysis of the 2015 Programme for International Student Assessment, Thomson, De Bortoli, and Underwood (2016) reported that Australia's performance in mathematical literacy declined significantly from 2009 to 2015. Within this concerning result, the proportion of high-performing Australian students in mathematical literacy from 2003 to 2015 decreased by 9%. Achievement at these top levels (the Advanced International Benchmark) reflects students' ability to apply knowledge as well as generalise in complex problem situations.

In their examination of Australia's performance in the TIMSS 1999 video study of Year 8 mathematics teaching, Lokan, McRae, and Hollingsworth (2003) noted a "widespread shallow teaching syndrome where the focus is on carrying out procedures without reasons" (p.

xxi). The report recommended “the inclusion of more challenging problems, less time spent on repetitive tasks, and an emphasis on higher-level reasoning and discussion to promote the connection between ideas” (p. xxi). Thomson and Fleming’s (2004) summary report on TIMSS 2003 expressed concern that 22% of Australian Year 8 teachers agreed or strongly agreed with the statement “learning mathematics mainly involves memorising,” with the researchers noting that this was a “learning strategy that rarely leads to deep understanding” (p. 86).

Concerns were also raised by Sullivan (2011) as part of the Australian Council for Educational Research’s education review *Teaching Mathematics: Using Research-Informed Strategies*. He reported that around 15% of Year 9 students were unable to answer a simple percentage question, and around 41% of the same students were unable to choose the correct response for a two-step fractions problem that involved an understanding of how to calculate common fractions and percentages of whole numbers. While taking care in interpreting these results, suggestions about the conceptual knowledge and reasoning skills of Australian students, and the provision of associated pedagogies in Australian schools, were put forward for consideration.

Recognising the increasing demands on Australian schools to attend to 21st century skills, including the ability to solve complex social and environmental issues, a working group of Australian education ministers met and agreed upon two overarching goals as part of the 2008 Melbourne Declaration on Educational Goals for Young Australians (Ministerial Council on Education, Employment, Training and Youth Affairs; MCEETYA, 2008). These goals focused on the provision of equity and excellence in schools so students could become creative, confident, and successful learners. In working towards these goals, it was proposed that schools needed to provide students with opportunities to "think deeply and logically" (p. 8) within a curriculum that supported "deep knowledge within a discipline" as well as "flexible and analytical thinking" (p. 13). The Declaration also committed to supporting quality teaching and school leadership and designing curriculum that would provide a foundation for interdisciplinary approaches to innovation and complex problem-solving.

The Melbourne Declaration's companion document was MCEETYA's four-year plan (2009). It committed to the establishment of ACARA to deliver rigorous, world-class national curriculum that would provide opportunities for students to reach the outlined goals. Through its implementation, Australia's first national mathematics curriculum (Australian Curriculum: Mathematics; AC:M) proposed that students enjoy access "to the power of mathematical reasoning" and opportunities to "apply their mathematical understanding creatively" through "in-depth study of critical skills and concepts" (ACARA, n.d.-a, "Rationale," para. 4). The proficiency strands of the AC:M (Understanding, Fluency, Problem-Solving, and Reasoning) describe "how content is explored or developed" (ACARA, n.d.-b). To emphasise the importance of learning content through such processes, these proficiencies are embedded in the syllabus content. This infers a pedagogical approach to teaching mathematics that has moved away from transmission teaching, where the teacher "transmits" knowledge to the students, towards a more flexible and interactive planning and pedagogical approach.

The expectation that teachers know their content and how to teach it is represented in a range of Australian and international professional standards documents. For example, the Australian Professional Standards for Teachers (Australian Institute for Teaching and School Leadership, AITSL, 2017) define the "knowledge practice and professional engagement needed for high quality effective teaching that improves student learning outcomes" (p. 10). Such standards act as public statements of what is valued and expected of teachers at different levels of expertise; they seek to define the desired traits of the teaching profession in order to achieve its goals.

The AITSL standards describe benchmark knowledge and skills of teachers across four key career stages: graduate, proficient, highly accomplished, and lead. Elements within these stages are separated into three key teaching domains: professional knowledge, practice, and engagement. In the professional knowledge domain, which includes the aspects of subject content and pedagogy, Standard 2 states teachers must “know the content and how to teach it. (AITSL, p. 11). At the Proficient level (achieved early in a teacher’s career), it is expected that teachers’ planning tasks for students are able to “apply knowledge and understanding of effective teaching strategies to support students’ numeracy achievement” (p. 11).

The Australian Association of Mathematics Teachers’ (AAMT) *Standards for Excellence in Teaching Mathematics in Australian Schools* (AAMT, 2006) stressed the need for connected knowledge and skills in its three domains: professional knowledge, attributes, and practice. These included being a “confident and competent user of mathematics who understand(s) connections within mathematics, between mathematics and other subject areas, and how mathematics is related to society” (p. 2).

In the US, documents such as the widely referenced National Council of Teachers of Mathematics’ *Principles and Standards for School Mathematics* (NCTM, 2000) and more recently the *Principles to Actions: Ensuring Mathematics Success for All* (NCTM, 2014) also reflected such expectations. In the latter document, under the principle of Teaching, it said that teachers must “know and understand deeply the mathematics they are teaching and be able to draw on that knowledge with flexibility in their teaching tasks” (p. 2). Under the guiding principle of Professionalism, it stated that U.S. teachers must “continually grow in knowledge of mathematics for teaching, mathematical pedagogical content knowledge, and knowledge of students as learners of mathematics” (NCTM, 2014, p. 116). These standards reflect the expectations of the teaching professional from the perspective of those employing teachers and those supporting them through their membership.

The statements from the US and Australia are consistent; teachers must know and understand the mathematics they teach and have sufficient flexibility with its content. They must have conceptual understanding.

Combined with my local context observations and school concerns about mathematical content knowledge (MCK) came increased expectations of the new AC:M, underpinned by its emphasis on problem-solving as a means to understanding mathematical content.

1.4 NEW SOUTH WALES CONTEXT

At the time the new Australian Mathematics curriculum (AC:M) was implemented in 2013, the NSW Department of Education (DoE) lost $201 million from its budget as part of a $1.7 billion four-year saving. This resulted in staff reduction across the DoE of 1,800 jobs, including 600 state and regional office staff. These consultancy staff had been pivotal in supporting schools to implement curriculum change. The cuts meant that such support had diminished to two mathematics personnel across the state, leaving schools to implement new mathematics curriculum without sufficient assistance from subject specialists. This realignment caused considerable concern at the school level; community rallies were organised by the NSW Teachers Federation as community days of action to reinforce the call for greater professional support. Despite such protests from teachers and schools, the cuts continued and professional learning responsibilities fell principally to the school context.

The NSW DoE's solution to calls for support became centrally designed online professional development courses that had to be adapted by school professional leadership personnel to meet the needs of their staff. This approach assumed that schools had sufficient expertise within them to make meaning of the knowledge presented. This situation was problematic for schools with staff who lacked deep subject matter knowledge and understanding and who did not have the funds to source the appropriate support. For schools with leadership that also had little deep understanding of syllabus content and ways to assist teachers with their conceptual understanding, this was even more problematic. The new

AC:M's emphasis on programming units of work that build the conceptual understanding of students presented challenges for teachers and leadership personnel with low conceptual understanding themselves.

1.5 RESEARCH PROBLEM

Considering the context outlined for the study, the research problem is stated as follows:

Many primary teachers, for whom mathematics is not a specialty, need support from professional development designed to promote increasingly sophisticated understanding and conceptual structure for themselves and their students. There are great demands on teachers' levels of knowledge of mathematics and teaching mathematics. Many teachers do not have the conceptual understanding to deliver the expectations of the AC:M (Ball, Hill, & Bass, 2005; Sullivan, 2011). They need support, but this is limited. State budget cuts have resulted in fewer personnel with the expertise to assist teachers and schools.

1.6 RESEARCH PURPOSE

The purpose of this research was to explore the impact of teaching an innovative mathematics curriculum on teachers' knowledge and beliefs about mathematics and mathematics teaching and learning.

1.7 RESEARCH QUESTION

There has been considerable research in the last 15 years about the components of knowledge required to teach mathematics effectively. It is increasingly clear that *how* teachers hold knowledge may matter more than *how much* knowledge they hold. My research question emanated from this proposal.

The major question underpinning the thesis is

How does implementing an innovative mathematics curriculum provide a stimulus for teacher professional learning?

1.8 SIGNIFICANCE OF RESEARCH

This research is significant because it has the potential to

- describe the complex nature of the belief structure and knowledge acquisition of practising teachers and the possible influences these have on classroom practice; and
- inform opportunities for meaningful professional development strategies, like the use of innovative curriculum, that have the potential to develop mathematical knowledge for teaching.

1.9 STRUCTURE OF THESIS

In Chapter 2, I examine the relevant research literature for my study. In particular, I focus on four related themes: teacher knowledge, teacher beliefs and attitudes, professional growth and change, and the mathematics curriculum. Research presented in relation to teacher knowledge about mathematics and its relationship to teacher practice addresses a range of conceptualisations, starting with the seminal work of Lee Shulman (1986). This analysis documents agreement that teacher knowledge of mathematics is important but that definition of such a term is complex. Research that supports the idea that teachers' beliefs about mathematics and about teaching mathematics and their important role in shaping classroom practice is also discussed in Chapter 2. As products of the system in which they then work,

teachers experience an "apprenticeship of observation" (Lortie, 1975, p. 61) from which they begin to form an understanding of their role in the classroom. The structural complexity of beliefs creates challenges for those attempting to analyse their effect on teacher practice and is one of the reasons offered in the literature for the range of findings in relation to the causality of beliefs and practice (Thompson, 1992). The role of context and the importance of reflection on teacher knowledge and beliefs are addressed as part of the research review on process of growth and change. Features of effective professional learning identified in the literature are compared in an endeavour to place the role of curriculum in this process.

In Chapter 3, the research design is outlined and justified in relation to the research purpose and question. Two Year 5/6 teachers were studied as they taught a rational number unit of work originally developed in the Netherlands and adapted for use in the US in a project called Mathematics in Context (MiC) (Britannica, 1997). Realistic Mathematics Education (RME; see, e.g., Streefland, 1991) promotes conceptual understanding using “models of” realistic contexts to develop “models for” abstract representations. Such heuristics are the basis of the *Some of the Parts* (Keijzer et al., 2006) unit selected for this research. A theoretical perspective of interpretivism was employed to promote in-depth understanding of each teacher’s world and their interpretations in this context (Merriam, 1998). Case study was the methodology adopted to best describe and understand the complex interactions and multiple realities of these teachers as they implemented curriculum in their mathematics classrooms. Ethics approval for this research can be seen in Appendix 1.

The interpretative design was viewed through a symbolic interactionism lens to acknowledge the centrality of significant symbols (like language and behaviour) as teachers developed meanings and defined their own reality (Crotty, 1998). Teachers ascribe meaning to what mathematics is, how it should be taught, and their role within this relationship. They also relate to the expectations of the mandatory AC:M and the demands of 21st century process skills that promote greater student interaction with content. To comprehend such activities as a researcher, it was incumbent upon me to attempt to perceive such objects and situations as the participants did (Charon, 2007). In this endeavour, the concept of identity was central. How teachers define themselves, their role in the classroom as a learner and teacher of mathematics, and their capacity for change are the products of the interaction they had, and have, with mathematics. To enter the case study teachers' community, I had to take on the role of a social object, observing the significant symbols of language and behaviour. Entering this role allowed me to take the standpoint of the teachers being studied and to generate dialogue that provided insight into how teachers defined their reality and the interpretations they made.

In Chapters 4 and 5, respectively, the classroom practices of case study teachers "Mark" and "Debbi" (pseudonyms) are described in detail, both before and during the innovative curriculum trial, with particular focus on any observable or reported changes in knowledge and beliefs during this period. In both these chapters, the main themes generated from the data are reported, with particular focus on how these themes related to the research purpose and question. The mathematical knowledge for teaching (MKT) model (Hill, Ball, & Schilling, 2008) and interconnected model of teacher professional growth (IMTPG; Clarke & Hollingsworth, 2002) were used together to analyse the kinds of mathematical knowledge teachers drew on when teaching an innovative unit of work and the effect (if any) on their classroom practice, beliefs, and attitudes. Modelling of this growth and change allowed insight into the affordances and constraints of such immersion strategies.

In Chapter 6, comparative discussion of Mark and Debbi’s responses to the innovative curriculum is made in relation to the research subquestions to contribute further analytical competence in relation to the overarching research question. There is a focus on several themes, with accompanying claims related to the data generated during the literature review and the innovative curriculum trial period. At the end of each section of the discussion, theoretical propositions are made with the view to addressing the major research question.

In Chapter 7, I restate the theoretical propositions related to the major research questions and the model of change underpinning the study, discuss implications for practice, acknowledge the research limitations, and make recommendations for further research.

Chapter 2: Literature Review

2.1 CHAPTER OVERVIEW

This study addressed the following research problem: Many primary teachers need support from professional development designed to promote increasingly sophisticated understanding and conceptual structure. There are great demands on teachers' levels of knowledge of mathematics and teaching mathematics. Many teachers do not have the conceptual understanding to deliver the expectations of the Australian mathematics curriculum (Ball et al., 2005; Sullivan, 2011). They need support, but this is limited; state budget cuts have resulted in fewer personnel with the expertise to assist teachers and schools.

This chapter addresses the theoretical and empirical literature underpinning the research problem. This literature review seeks to fulfil the following goals:

1. Demonstrate a familiarity with a body of knowledge and establish credibility.
2. Show the path of prior research and how the current project is linked to it.
3. Integrate and summarise what is known in an area.
4. Learn from others and stimulate new ideas. (Neuman, 2000, p. 111)

As indicated in Chapter 1, the purpose of this research was to explore the impact of teaching an innovative mathematics curriculum on teachers' knowledge and beliefs about mathematics and mathematics teaching and learning. To ascertain what is already known,

research and professional literature were sourced from peer-reviewed journals and books

produced by esteemed academic research publishers and professional associations via library

database searches. Such an approach located high-quality research literature designed to inform

the specialist fields to which they are directed, contributing to its esteem and indicating

worthiness of inclusion.

Literature searches were conducted in a variety of ways and across a range of databases.

Seminal articles written by key researchers in relevant fields up to 30 years ago were included

due to their continued reference in current research articles. Journal articles were identified

through keyword searches of the broad-reaching Education Resources Information Center

(ERIC) database. To add to the breadth and depth of this search, the assistance of the ERIC

thesaurus was accessed. This feature provides alternative search terms the database may use for

the descriptors provided in the search. Keywords were used from the research purpose with

limiters that restricted articles to the past 20 years, then the past 10, five, and one year. Filters

restricted to accessing articles written in Australia were also used to find the research articles

written about research in Australian schools. Highly referenced authors in these searches were

also noted to identify those researchers who are leaders in the field. Literature specific to mathematics education research was also accessed by way of keyword searches that the university-authorised servers deemed sufficiently respectable. Ancestry searches from relevant articles, chapters, and books were conducted and literature advice sought from supervisors who are also current researchers in mathematics education.

Upon systematically reading the literature identified in initial and subsequent searches, summaries were created to record research questions, data generation procedures, and findings. Notes were made about the scope and possible significance of the literature read in relation to my proposed study. A "visual rendering" (Creswell, 2008, p. 107) process was conducted by way of a literature map to assist me to see visual overlaps in information and to generate themes. This process assisted in the creation of a concept map driven by four themes or content areas relevant to my study. This map also assisted me to ascertain how my research would add to or extend the existing literature. A refined concept map resulted (Figure. 2.1), providing a framework for the analysis of the body of literature relating to the MCK of teachers, their beliefs and attitudes about mathematics and teaching mathematics, and effective professional learning strategies. Acknowledging the important role of context, this conceptual framework is embedded in the mandatory work and expectations of teachers in relation to curriculum implementation in the classroom setting.

Figure 2.1. Conceptual framework of the literature review.

This conceptual framework of the literature supports the structure of this chapter. Section

2.2 addresses teacher knowledge of mathematics and teaching mathematics, Section 2.3

addresses teacher beliefs and attitudes about mathematics and the teaching of mathematics,

Section 2.4 addresses literature on the professional growth and change of teachers, and Section

2.5 embeds the previous themes into the wider context of Australian mathematics curriculum

expectations. The themes of knowledge, beliefs, and professional learning are connected in

analysis of the role curriculum support materials have played in promoting teacher learning.

In the following section, I discuss teacher knowledge about mathematics and its

relationship to teacher practice. The historical context presented frames its current counterpart,

with a view to identifying and describing important themes that underpin the research problem

and purpose.

2.2 TEACHER KNOWLEDGE OF MATHEMATICS AND TEACHING MATHEMATICS

The importance of teacher knowledge and understanding of subject matter and related

pedagogies is a seemingly unchallenged tenet in the research literature over time (Ball Thames,

& Phelps, 2008; Ma, 1999; Shulman, 1987; Skemp, 1976; Stein, Remillard, & Smith, 2007).

There is still variation, however, in what is considered teachers’ mathematical knowledge

(Copur-Gencturk, 2015). Those studying the influence of knowledge and understanding on

pedagogy have attempted to delineate and define the complexities associated with and between

these aspects of teaching. This has led to the development of a range of conceptual frameworks

that describe the components of knowledge required to teach mathematics effectively (Ball et al., 2008; Chick, Baker, Pham, & Cheng, 2006; Fennema & Franke, 1992; Petrou & Golding, 2011; Tchoshanov, 2011). This review discusses some of these frameworks in detail because of their continued prominence in current mathematics research literature and because they form the basis from which Australian researchers continue to draw. The literature in this section is addressed under the following subsections: conceptualising and categorising knowledge, attempts to assess teachers' knowledge in mathematics, further work on frameworks of knowledge and the interrelationships of categories, teacher knowledge and understanding of content, and the relationship between knowledge and student achievement. As the research literature in relation to teacher knowledge across all subject areas is vast, the literature addressed is narrowed to the field of mathematics education. More general research is only cited when seminal.

2.2.1 Conceptualising and categorising knowledge

In recent years, a large body of research has used the theoretical basis of Shulman's (1986) content knowledge model to explore the kinds of knowledge teachers bring to the classroom. Researchers across subject areas continue to refer to Shulman's foundational publications about an amalgam of subject matter knowledge and pedagogical knowledge, with more than 19,950 publications citing his 1986 article (Google Scholar). Landmark studies used this categorisation to explore the depth and extent of teacher knowledge (Ma, 1999), the nature of such knowledge (Askew, Brown, Rhodes, Wiliam, & Johnson, 1997), and the pivotal importance surrounding knowledge for teaching mathematics (Borko et al., 1993). Though widely supported, some researchers criticised the “static” nature of Shulman’s model (e.g., Fennema & Franke, 1992; Hasweh, 2005), both in its categorisation and the nature of where knowledge growth is situated (out of the context of the classroom). Others extended and explored the model, considering examples of the ways content and pedagogy change according

to their context (Chick, Pham, & Baker, 2006). For some researchers, Shulman’s conceptualisation did not make distinctions between subject matter knowledge (SMK) and pedagogical content knowledge (PCK) sufficiently clear (see, e.g., Ball et al., 2008; Hill et al., 2008). These scholars reorganised and extended his work to include subcategories specific to teaching mathematics effectively. I now examine some of these studies to explicate the various approaches to categorising teacher knowledge and justify the need for further study of this complicated construct.

The seminal work of Shulman (1986, 1987) and his inquiry into the sources of teacher knowledge was timely in the prevailing public analysis of knowledge and pedagogy driven by calls to improve standards in mathematics education (e.g., Committee of Inquiry into the

Teaching of Mathematics in Schools, 1982; National Commission on Excellence in Education, 1983; National Science Board Commission on Precollege Education in Mathematics, Science and Technology, 1983). Presented as a generalist proposition, not specific to a particular subject area, Shulman (1987) categorised the content knowledge needed for teaching. The seven categories included

- general pedagogical knowledge;
- knowledge of learners' characteristics;
- knowledge of educational context;
- knowledge of educational purposes and values;
- content (subject matter) knowledge;
- pedagogical content knowledge;
- curriculum knowledge. (Shulman, 1987, p. 8)

The first four categories dealing with general aspects of teacher knowledge were not the main focus of Shulman's work. The last three categories are content dimensions and made up what Shulman termed the "missing paradigm" (Shulman, 1986, p. 7) in teacher research.

Shulman identified PCK of special interest because it "represents the blending of content and pedagogy into and understanding of how particular topics, problems, or issues are organised, represented, and adapted to the diverse interest and abilities of learners, and presented for instruction" (1987, p. 8).

In explaining the starting point of comprehension, he referred to a critical expectation of those who teach:

> To teach is to first understand. We ask that the teacher comprehend critically a set of ideas to be taught. We expect teachers to understand what they teach, and when possible, to understand it in several ways. They should understand how a given idea relates well to other ideas within the same subject area and to ideas in other subjects as well. (Shulman, 1987, p. 14)

Content knowledge, also described as SMK (Shulman, 1986), aligns with the relational and conceptual knowledge descriptions of Skemp (1986) and Hiebert and Lefevre (1986), respectively. Shulman described the need for substantive and syntactic knowledge (Schwab, 1978) of subject matter, as well the ability to define and explain across and within a subject's structure. The second of these categories, PCK (Shulman, 1986), describes the critical intersection of content knowledge *for* teaching and *of* teaching. Such knowledge includes "the most useful forms of representation of those ideas, the most powerful analogies, illustrations, examples, explanations and demonstrations that make them comprehensible to others" (Shulman, 1986, p. 9). It also involves knowledge of what makes the learning of particular topics easy or difficult and ways to cater for the conceptions, preconceptions, and misconceptions that students bring with them to school. His last content-related category, curricular knowledge, includes knowledge and understanding of the range of curricular alternatives designed to teach a particular subject (horizontal knowledge), including knowledge of what came before and will come after a year level topic (vertical knowledge).

Ball (1988) questioned the assumptions made about (prospective) teachers' SMK, the focus on pedagogy and skills, and reliance on a teacher's own schooling history as sufficient "knowledge" to teach mathematics. This questioning funnelled into two influential future research veins: a teacher's knowledge *of* and *about* mathematics. Like Shulman, Ball argued that knowledge of mathematics is fundamental to being able to teach it to someone else. She argued that "knowing mathematics flexibly or in depth" (p. 9) is underconceptualised. Ball's focus on the nature of knowledge in the discipline of mathematics explored similar understandings referred to by Schwab (1978), knowledge of the substantive and syntactic structure of the discipline.

Much of the early research to probe teachers' knowledge was used with preservice teachers due to the ease of accessibility for educational researchers (in universities), as well as problems surrounding the ethical issues of "testing" in-service teachers. Posing mathematical tasks and following up with interviews to discuss reasoning was a common methodology. One of the limitations of this approach is its distance from the actual practice of teaching. It begins with the knowledge and the teacher separately—not the connection between them, knowledge in *teaching*.

2.2.2 Attempts to assess teachers' knowledge in mathematics in context

Situated theorists argue the embedded nature of knowledge and its location in social practice (Lave & Wenger, 1991). The dynamic process of *knowing* is seen as an active description of the application of *knowledge*, its static, abstract, and passive form (Adler, 1998). Adherents to this theoretical view see any attempt to describe knowledge out of the context of the classroom as problematic. In light of this contextual perspective, researchers have observed teachers and their application of mathematical knowledge *for* teaching in the classroom (Carpenter, Fennema, Peterson, & Carey, 1988; Eisenhart et al., 1998). The results of such research have emphasised the importance of PCK and stimulated further interest in categorising and conceptualising teacher knowledge.

Modelling by Fennema and Franke (1992) recognised the interactive and dynamic nature of knowledge, in particular the interrelationships that occur in the context of the classroom while teaching. They include content of mathematics, pedagogical knowledge, knowledge of learners' cognitions in mathematics, and beliefs. The model's content component has visible association with Shulman's SMK in its emphasis on conceptual understanding of content and the interrelatedness of mathematical ideas. New knowledge is created in the teaching process.

Teachers draw on their pedagogical skills and make decisions while interacting with subject

matter and students. Shulman (1987) also acknowledged this transformation of a teacher's

complex knowledge base into accessible parts in his cycle of pedagogical reasoning. Such

transformation is complex and tied to the context in which it is developed. If the context changes

(different content, classroom structure, students), the knowledge the teacher draws on will also

change. Pivotal to these interchanges of knowledge is the complexity of the knowledge base

the teacher has to draw on (depth of SMK) and its interconnections. Examples of such

interaction were documented by the Cognitively Guided Instruction project team (Carpenter,

Fennema, Peterson, & Carey, 1992), in particular the difference in classroom behaviour of

teachers whose general pedagogical knowledge was observably limited when the mathematical

content of lessons changed. This difference in classroom behaviour (the ability to transform

knowledge) was partly due to the teacher's depth of content knowledge and its connecting ideas

and the beliefs that she held about her ability to teach the concept of fractions (Lehrer & Franke,

1992). Askew et al. (1997) also recognised the central role of teachers' practice when modelling

the interplay and relationship between beliefs, knowledge, and classroom practices. Developed as a result of the Effective Teachers of Numeracy review in the UK, questionnaires generated data from 90 teachers about their organisation and planning for mathematics teaching, use of

resources, training and continuing professional development, and perception of teaching styles. It also generated data on these teachers’ knowledge and beliefs with regard to numeracy and their beliefs about teaching, learning, and assessing mathematics in general. From these 90 teachers, 18 case study teachers were chosen to be observed over two school terms, allowing researchers to gather data on classroom practices and knowledge and beliefs about mathematics, students, and teaching. Because the project used the definition of highly effective teachers as those who positively influenced student outcomes (achieving a higher average gain in numeracy in comparison with other classes from the same year group), the model developed placed teachers’ practice in the centre. The strongest effects on teacher practice were found to be teachers’ PCK and teachers’ beliefs.

This research was an important contributor to the increasing knowledge base about knowledge in teaching. Using the term and premise behind Shulman’s PCK, it incorporated detailed observations and analysis of teachers’ application of knowledge in context, as well as the ability of teachers to make connections across knowledge bases (SMK). In their discussion,

Askew et al. (1997) suggested that the nature of the knowledge about the subject matter teachers

had, rather than the level of formal qualifications they acquired, was pivotal in their ability to

connect ideas, representations, and explanations. Highly effective teachers, they concluded, are

more likely to have a *connectionist* orientation. This orientation assists such teachers to develop

their own conceptual basis for numeracy strategies, using discussion and challenge to introduce

links between different meanings and representations.

The centrality of classroom practice that applies a multifaceted and connectionist

approach to teaching was also addressed in the research of Ma (1999) who contributed

important cross-cultural perspective to the notion of SMK and PCK. This research documented

the differences between Chinese and U.S. teachers' content knowledge through hypothetical

classroom scenarios presented to 72 Chinese and 23 U.S. primary teachers. The scenarios were

designed to assess the presence of a flexible and dynamic knowledge of content (SMK),

identified by Shulman (1986) as an essential component of knowledgeable teaching. The

framework used included understanding of basic ideas, connectedness, multiple

representations, and longitudinal coherence. This conception of mathematical understanding,

Shulman wrote in the foreword (Ma, 1999), emphasised the aspects of knowledge most likely

to contribute to a teacher's ability to explain mathematical ideas to students to transform it. In her final observations, Ma concluded that Chinese teachers seemed to have a “profound understanding of fundamental mathematics” (PUFM; p. 118). They were able to draw a deep and thorough understanding of mathematics as well as being able to represent the connections

between mathematical ideas. Such “knowledge packages” (p. 18) suggested that subject matter of Chinese teachers was strong and flexible.

2.2.3 Framework refinements and interrelationships of knowledge categories

Interest in the interaction of knowledge of content and pedagogy and the contexts in which they were observable led to the most recent elaborations of the PCK construct. Using a range of questionnaire-then-interview methods and in-class video analysis, researchers further investigated relationships between components of teachers’ knowledge. The nature of teachers’ knowledge was still of interest, not just what teachers knew but how they knew and when they used it in the course of their work.

In Australia, Chick, Baker, et al. (2006) developed a framework for analysing PCK in three categories: Clearly PCK, Content Knowledge in a Pedagogical Context, and Pedagogical Knowledge in a Content Context. The first category, Clearly PCK, involved what they termed the “inseparable components of pedagogy and content” (p. 298), including teaching strategies,

student thinking, alternative models or representations, and resources and curriculum. The

second category, Content Knowledge in a Pedagogical context, focused on the mathematics

needed for teaching. It included PUFM (Ma, 1999), deconstructing knowledge into key

components, and highlighting mathematical connections. This category acknowledged the

presence of procedural knowledge. Like Hiebert and Lefevre (1986), the authors conceded that

such knowledge need not be tied to conceptual understanding at its basis. The final category,

Pedagogical Knowledge in a Content Context, included generic teaching knowledge, much like

the aspects of Shulman's (1987) general pedagogical content classification.

Chick, Baker, et al. (2006) acknowledged the overlap among the aspects within and across

categories. The characteristics of the framework, they argued, allowed a focus on deep

distinctions in PCK in relation to 14 Australian Year 5/6 teachers' understanding of complex

topics like decimals. A take-home questionnaire then follow-up interview method was used to

allow teachers to think about their responses before being asked to justify them. These

composite answers were then analysed using the PCK framework developed, while still under

trial. The researchers were attempting to assess what PCK teachers brought to the topic of

decimals and if this knowledge was sufficient. One particular item, designed to focus on one

aspect of their framework, elicited 348 different instances of PCK, alluding to the complexity of teaching and the knowledge required, in particular the degrees of PUFM.

In an effort to establish a clearer definition for the concepts of SMK and PCK through empirical testing, Ball and her colleagues (2008) proposed a framework of mathematical

knowledge for teaching (Hill et al., 2008). This widely referenced conceptualisation applied the acknowledgement made by Shulman to Boaler (2003), and other researchers in the field, that more emphasis in his model was needed on teacher action in practice and teacher learning. Ball et al. (2008) reorganised and extended the theoretical aspects of Shulman's SMK and PCK that focused on the "work of teaching," specifically *how* teachers needed to know content, *what else* they needed to know about mathematics, and where they might *use* such mathematical knowledge in practice (their emphasis). Through extensive qualitative analysis of videos of teaching practice, and research designed to measure this knowledge base, they developed a working definition of mathematical knowledge for teaching: "the mathematical knowledge that teachers need to carry out their work as teachers of mathematics" (Ball et al., 2008, p. 4).

The resultant framework (see Figure 2.2) divided Shulman's conceptualisation of SMK into three strands: common content knowledge (CCK), horizon content knowledge (HCK) and specialised content knowledge (SCK). It also separated Shulman's PCK into three strands:

knowledge of content and students (KCS), knowledge of content and teaching (KCT), and

knowledge of content and curriculum (KCC). CCK includes the knowledge most educated

adults would have, for example, whether what is in a mathematics curriculum or textbook is

accurately described and being able to use mathematical terms and notation correctly. Horizon

content knowledge (HCK) is an awareness of how mathematical ideas are related to the span

of the curriculum. SCK includes how to represent mathematical ideas, explanations for common

rules and procedures, and how to examine and understand different methods to problems. KCS

includes the ability to anticipate what students will find hard or easy, how best to build on

students’ mathematical thinking, and how best to address student errors. KCT is an

understanding of how to sequence particular content for instructions, how to evaluate

instructional advantages and disadvantages of particular representations, and to make

instructional decisions about which student discussion contributions to pursue, ignore, or save

for later use. KCC is knowledge of instructional materials available for particular teaching

topics and why these may be appropriate or inappropriate to use (Ball et al., 2008).

Ball et al. (2008) recognised the subtlety of the lines between the types of knowledge in

the model, in particular CCK, SCK, KCS, and KCT. A teacher who recognises a wrong answer

may be drawing on their CCK then using their SCK while working out the nature of the error.

They may then draw on their KCS to determine why that particular student failed to carry out the problem correctly (the individual's age/ability or knowledge related to teaching this concept) and the appropriate way to address it (KCT).

Figure 2.2. Comparison of Shulman's (1986) original category scheme and Ball et al.'s (2008) categories of mathematical knowledge for teaching (MKT) (p. 403).

Of particular interest to the authors was evidence related to SCK, which Ball et al. (2008) and Hill et al. (2008) hypothesised was closely related to practice and mathematical knowledge but beyond that of the average person, and different to that of a mathematician. This new and relatively unchartered conceptualisation distinguished the complex SMK needed by teachers in the course of their work.

In testing the conceptualisation of their model, and in particular KCS, Hill et al. (2008)

developed pre and post assessments for teachers involved in California's Professional Development Institute programs in mathematics. Each assessment contained multiple-choice items to assess both CCK and SCK across a range of number, patterns, and algebra domains, as well as KCS (in relation to number only). Cognitive interviews then formed part of the validity process to help determine whether item design did actually measure KCS as conceptualised in the MKT model. In their discussion and conclusion, Hill et al. (2008) reflected on the complexity of the very notion of "knowledge." They considered that skill distinction between "knowing" that a child has difficulty with a particular topic and "reasoning" why that is so and how to best address it are different dimensions of the same KCS category.

In working out why students had made an error, some teachers analysed the problem using mathematical reasoning, then related this to what the students had done. Others leant more heavily on their knowledge and experience with such student errors. Though indicating that conceptualisation of such a domain is far from straightforward, this finding suggested that teachers do use knowledge of content and knowledge of students together to analyse student responses to a mathematical problem.

Petrou and Goulding (2011) highlighted the uncertainty expressed by Ball et al. (2008) about whether their PCK category of knowledge of content and curriculum should be part of several categories or one in its own right (see Figure 2.3). Similar questioning was expressed about HCK, categorised by Shulman as part of CCK (in particular the vertical curriculum).

Petrou and Goulding's theoretical model arose from the investigation of the relationship between preservice primary school teachers' SMK and PCK of mathematics. Detailed analysis of observations and videorecorded mathematics lessons taught by preservice teachers in a one-year postgraduate certificate course resulted in classification of knowledge that was enacted in the classroom. The researchers made particular reference to and synthesised the models and theories of Shulman (1986), Schwab (1978), Fennema and Franke (1992), Rowland, Huckstep, and Thwaites (2005), and Ball et al. (2008).

Although not part of the original modelling of teacher knowledge in action, the importance of curriculum knowledge was later highlighted in understanding what teachers need to know to teach mathematics effectively. Petrou and Goulding's model also implied that teachers' SMK and PCK can determine the ways teachers understand, interpret, and use the curriculum and vice versa. In light of this, Petrou and Goulding (2011) called for further research about how teachers use the curriculum to improve their teaching and which materials are most effective in doing this.

Figure 2.3. Synthesis of models on teacher mathematical knowledge (Petrou & Goulding, 2011, p. 21).

A framework of PCK aimed at "illuminating knowledge used in the practice of teaching"

(p. 660) was developed by Roche and Clarke (2011) as part of the two-year Contemporary

Teaching and Learning of Mathematics program, involving 82 Catholic primary schools in

Victoria. Participating teachers' PCK was assessed at the beginning and end of a particular year

of learning and results assessed to determine growth in PCK. The framework included

- Pathways: Understanding possible pathways or learning trajectories within and across mathematical domains, including identifying key ideas in a particular mathematical domain;
- Selecting: Planning or selecting appropriate teaching/learning materials, examples or methods for representing particular mathematical ideas including evaluating the instructional advantages and disadvantages of representations or definitions used to teach a particular topic, concept or skill;
- Interpreting: Interpreting, evaluating and anticipating students' mathematical solutions, arguments or representations (verbal or written, novel or typical), including misconceptions;
- Demand: Understanding the relative cognitive demands of tasks/activities;
- Adapting: Adapting a task for different student needs or to enable its use with a wider range of students. (p. 659)

The questionnaire approach used in this research elicited information through open responses, creating some methodological difficulty in relation to coding reliability. The researchers acknowledged that a more comprehensive view of teachers' knowledge would have been obtained through interview protocols and direct observation of practice, suggesting the need for a case study approach. The difficulty of defining and assessing the complex nature of teacher knowledge was an important finding in this research.

Common to the models and frameworks presented is acknowledgment of the dynamic nature of MKT. Some models present categorisations in a static form with suggestions from experience and research of overlap and interaction; others use arrows to indicate relationships and interplay between categories of knowledge. The ambiguous boundaries of SMK and PCK are discussed regularly in the research. Such delineations may appear artificial but are useful in driving research that attempts to deconstruct the multifaceted nature of teacher knowledge and talk about the interconnections between domains (Ball et al., 2008). Unlike Fennema and Franke (1992) and Askew et al. (1997), Ball et al.'s (2008) conceptualisation of teacher knowledge did not include the influence of beliefs on mathematics teaching. Petrou and Goulding (2011) embedded beliefs as part of SMK. Askew et al.'s (1997) modelling of beliefs is related to students and learning, but the model of Fennema and Franke (1992) suggested (as did examples from their research) that beliefs also hold status over a teacher's conception of themselves as a practitioner. It is clear that defining and codifying teacher knowledge is as

complex as the construct it attempts to describe, and more recent calls for a deeper understanding of what teachers know and do when teaching mathematics (Barton, 2009; Beswick, 2011) are still relevant.

2.2.4 Teacher knowledge and understanding of content

There is a widely held assumption and public expectation that teachers both know and understand the content they are teaching. Current teaching expectations that emphasise promotion of mathematical proficiency, including adaptive reasoning, strategic competence, procedural fluency, and conceptual understanding, are articulated in the AC:M (ACARA, n.d.-b). The expectation that teachers implement programs designed to develop understanding in students places a focus on the connectedness of teachers' PCK and SMK. However, teachers who do not understand the content they are teaching are more likely to plan, teach, and assess such content in a narrow and inflexible manner (Shulman, 1987).

Skemp (1976) argued the difference between *instrumental* and *relational* understanding. In his view, the former involves "rules without reasons," whereas the latter promotes "knowing what to do and why" (p. 89). Relational understanding of mathematics builds up schemas or conceptual structures that allow flexible and independent investigation of a range of problems.

Mathematics taught instrumentally has the advantage of being easier to teach, the results are more apparent (observable), and the right answer can be achieved quickly. Mathematics taught relationally is harder to teach as it involves more actual content and requires relationships to be actively built up within this content. Learning mathematics with understanding has the benefits of being generative, promoting retention, reducing the amount that must be remembered, and enhancing transfer (of knowledge to new situations). The constructive consequences of such understanding extend to teachers and students (Hiebert & Carpenter, 1992).

The importance of a connected knowledge base was reported in the UK's *Effective Teachers of Numeracy* review (Askew et al., 1997) discussed earlier. An understanding of teachers' subject knowledge (its fluency and scope, as well as the links, depth, explanations, and understanding teachers could show in relation to this knowledge) was gained through questionnaire data, profiles, and observations. This detailed in-situ observation allowed access to contextualised knowledge: the use of content knowledge in planning and teaching. Follow-up concept-mapping interviews, during which teachers were asked to propose mathematical ideas they considered to be important in numeracy and show how these concepts (supplemented where necessary) were linked together, revealed information on a teacher's content knowledge

and knowledge of relationships. This qualitative data revealed that many teachers found it

difficult to discuss links between mathematical concepts they were teaching, especially in

relation to fractions and decimals. Observations of classroom practice also suggested that some

teachers were providing a very limited and disorganised view of mathematics and were unable

to satisfactorily link various parts of the curriculum. Askew et al. (1997) concluded that primary

teachers needed to develop a fuller and deeper understanding of numeracy to teach it

effectively.

Ma's (1999) research investigated U.S. and Chinese teachers' understanding of

fundamental mathematics. Her description of "knowledge packages" aligns with Hiebert and

Carpenter's (1992) web metaphor and Askew et al.'s (1997) connectionist emphases. Most of

the Chinese teachers interviewed by Ma used a pack or group of mathematical pieces

(knowledge and ideas) around a key concept as a way of connecting important information

when teaching mathematics. The difference in awareness of such connections, and the level of

organisation in the structure of the *knowledge package* (the conceptual basis of the topic being

taught), was associated with differences in teachers' subject knowledge and how to teach it.

Teachers with "unconscious packing of knowledge" were "vague and uncertain of the elements and the structure of the network" (p. 21). This led to procedural teaching approaches that lacked the explicit connections needed to promote student learning.

Teachers' understanding of mathematical representations to promote student understanding has been addressed in Australian research related to the use of tasks. The Task Types and Mathematics Learning project conducted a survey to investigate the use of mathematical tasks in Years 5 to 8 (67 primary and 40 secondary teachers) in Victorian schools. Teachers were asked to complete a survey focusing on their use of tasks. In Sullivan, Clarke, and Clarke (2013), the four types of tasks were described:

> - Type 1: Teacher uses a model, example, or explanation that elaborates or exemplifies the mathematics;
> - Type 2: Teacher situates mathematics within a contextualised practical problem to engage the students, but the motive is explicitly mathematics;
> - Type 3: Teacher poses open-ended tasks that allow students to investigate specific mathematical context;
> - Type 4: Teacher poses interdisciplinary investigations in which the assessment of learning in both mathematical and non-mathematical domains is possible. (sp. 87)

The subcategories of Ball et al.'s (2008) conceptualisation of MKT were used to summarise and interpret the responses of teachers. One of the survey prompts categorised as Type 1 presented a lesson idea question: Which is bigger: 2/3 or 201/301? Researchers found that most teachers were able to state the correct answer but had difficulty reasoning why. The

prompt that followed asked what mathematics they hoped the students would learn if they

developed a lesson based on this question/idea. Results analysed in relation to this specific item

of the survey found more than three quarters of the primary teachers were unable to identify the

focus or potential of this task, suggesting limited CCK and SCK. A related prompt that asked

teachers to describe a lesson they might teach based on this idea is a clear aspect of PCK. Their

findings suggested that many teachers found translating the fraction comparison task into a

worthwhile learning experience difficult or found it difficult to explain how they would do so

(KCC). Such research highlights the importance of all six MKT domains to design and

implement lessons effectively.

2.2.5 Relationship between knowledge and student achievement

A number of researchers have successfully documented the long-suspected relationship

between the mathematical knowledge of teachers and student achievement (Bobis, Higgins,

Cavanagh, & Roche, 2012; Hill, Rowan, & Ball, 2005; Rowland & Ruthven, 2011). In his meta-

analyses of more than 50,000 research studies on the major sources of variance in student

achievement, Hattie (2003) concluded that what teachers know, do, and care about is the second greatest source of variance after student innate ability. He referred to the deep representations about teaching and learning of expert teachers, the way their knowledge is used in the classroom, and the positive effect this PCK has on the depth of student processing.

Early research attempting to empirically establish the mathematical knowledge needed for teaching, and therefore student learning, tended to count the number of courses taken, credits achieved, and degrees attained and relate this back to students' learning. The use of proxy variables like number of courses attended, level of mathematics studied, and/or results of basic skills tests as predictors of student achievement takes a narrow view of the knowledge teachers need and how they interact with such knowledge in the process of teaching. The common maxim "more is better" is not fully supported by research findings (Beagle, 1979; Monk, 1994; Youngs & Qian, 2013). SMK imbued by mathematics courses makes a difference to student learning only up to a point. The compacting of ideas at high levels of mathematics, and encouragement of more procedural approaches to problem-solving in such courses, can negatively affect a teacher's ability to break down elementary mathematics into its fundamental parts (Ball, 1988; Ball & Bass, 2000).

The U.S. Department of Education's *Final Report of the National Mathematics Advisory Panel* (Flawn, 2008) reflected detailed analysis of relevant information and research in the following areas: conceptual knowledge and skills, learning processes, instructional practices, teachers and teacher education, and assessment. Its findings also found that higher teacher certification in mathematics was not necessarily a guarantee of higher mathematical gains for students. It went on to qualify this observation in its recommendation: “Teachers must know in detail the mathematical content they are responsible for teaching and its connections to other important mathematics, both prior to and beyond the level they are assigned to teach” (p. 37).

Hill et al. (2005) studied the effect of knowledge and understanding on teaching and learning opportunities provided to students. They attempted to empirically establish the influence of knowledge on student outcomes as part of the Study of Instructional Improvement (2000–2004). The complexity of teaching, lack of agreement about what constituted SMK, and ways to link these to improvements in student achievement caused difficulties for the researchers involved. Their assessment was designed to determine the actual mathematical content that teachers teach, as well as the SCK teachers needed for the work of *teaching*. Researchers collected survey (questionnaire and teacher logs) and student data (student assessments and parent interviews) from 115 elementary schools during two school years (2000–2001 and 2003–2004).

The teacher log served as a highly structured self-report instrument designed to gather information on the time, content, and instructional strategies provided to students in the study.

There were five and 12 items designed to measure teachers' content knowledge for teaching mathematics (CKT-M) on each questionnaire. They included questions designed to highlight two key elements of content knowledge needed by teachers: CCK (that of the educated public) and specialised content knowledge used specifically by teachers in classrooms. This specialised knowledge includes representations and explanations, and how to determine the validity of alternative solution methods. A range of content areas and difficulty levels were provided.

There was a clear indication that teachers' CKT-M was a significant predictor of student gains at both grade levels. The effect roughly translated to one half to two thirds of a month additional growth per standard deviation difference on the CKT-M variable, CKT-M being the strongest teacher-level predictor of student achievement.

Accordingly, Hill et al. (2005) suggested knowledgeable teachers "can positively and substantially affect students' learning of mathematics" (p. 396). They went on to note that the positive effect on student gains in Grade 1 suggests that teachers' content knowledge plays an important role "even in the teaching of very elementary mathematics content" (p. 399). This observation supports Ma's (1999) belief that primary mathematics is not superficial "and

anyone who has to teach it has to study it hard to understand it in a comprehensive way" (p. 146).

The importance of teachers having connected mathematical knowledge is clearly outlined in the research presented. The suggestion is that such connected knowledge affects the way a teacher teaches mathematics and therefore students' understanding. Ball et al.'s (2008) elaborations into the MKT domain were developed to enable researchers to ascertain which strands were the greatest predictors of student achievement and how this knowledge might affect efforts to improve teacher's content knowledge, either through teacher education and professional training or in the design of support materials.

2.2.6 Summary of literature on teacher knowledge

The findings of the research studies presented have contributed to an understanding of the mathematical knowledge needed for teaching and justified the need for such knowledge to be deep and connected. Seminal literature has attempted to categorise and analyse the types of knowledge and in doing so suggested a complexity in relation to the knowledge needed to teach mathematics for conceptual understanding. This complexity provides room for more research that attempts to understand the relationship between MKT domains and how they present themselves in a teacher's practice.

Although the models of Chick, Baker, et al. (2006) and Petrou and Goulding (2011) continue to be used by researchers, recent applications of these models tend to have been applied to preservice education teachers (Rowland, Turner, & Thwaites, 2014), other subject areas (e.g., outdoor teacher education) (Dymnent, Chick, Walker, & Macqueen, 2018), and teacher educators (Muir, Fielding-Wells, & Chick, 2017). The Ball et al. (2008) MKT model is most prominently applied when analysing practising mathematics teachers' knowledge because of its strong empirical base linking MCK to student achievement (e.g., Hill et al., 2008). Instruments used to capture the subject matter component of the MKT model are widely used by researchers investigating the impact of teacher professional development programs (Abt Associates, 2010, 2013; Copur-Gencturk, 2015). The MKT model does not acknowledge the role of context or beliefs, which may be seen as a constraint; however, it does fit the purpose of this research as its examples of the categorisation of knowledge are more practically exemplified than others included in this chapter.

Literature in this section highlighted the growing awareness of the effect teacher beliefs and attitudes have on teacher knowledge and practice. This interplay and its associated complexities are addressed in the following section on teacher beliefs and attitudes.

2.3 TEACHER BELIEFS AND ATTITUDES

Early researchers and theorists interested in the relationships between teacher beliefs and actions and student learning attempted to define and categorise the structural features of beliefs to create some common ground on which to compare findings. This seminal research literature (Ernest, 1989; Green, 1971; Nespor, 1987; Pajares, 1992; Raymond, 1997; Thompson, 1984, 1992) continues to be referenced by current researchers investigating mathematics education and the affective domain. Studies related to teacher beliefs in the last 30 years appear to investigate the impact of preservice teacher courses (Frykholm, 1999), causes and effect of anxiety (Stoehr, 2017; Harper & Daane, 1998), beginning teachers' beliefs and practices (Raymond, 1997; Skott, 2001), theoretical constructs that focus on beliefs (Anderson, White, & Sullivan, 2005; Ernest, 1989), the effect of an emphasis on nontraditional teaching and learning (Stipek, Givvin, Salom, & Macgyvers, 2001), the role of beliefs in a change environment (Handal & Herrington, 2003), and/or the effect of context on beliefs and belief systems (Ma, 1999; Perry, Wong, & Howard, 2006). Recent studies investigating the relationship between beliefs and knowledge have generally involved middle and high school teachers (Beswick, 2011). Some researchers have analysed teachers' beliefs in relation to changes in practice, in particular the shift to a problem-solving and inquiry-oriented approach to teaching mathematics (Anderson et al., 2005; Handal & Herrington, 2003; Skott, 2001). The relationship between beliefs and knowledge, and their congruent development in relation to curriculum reform, has been raised and highlighted as an important area of research (Beswick, 2011; Handal & Herrington, 2003; Wilkins, 2008).

An ERIC search of peer-reviewed research using the terms *teachers, beliefs, mathematics,* and their associated terms narrowed down to primary (elementary) teachers over the past five years resulted in only 40 publications. More than half the studies sourced in this search involved preservice teachers. Removing those not specifically focused on teaching mathematics (e.g., measurement scale development, robotics game design) resulted in 13 studies related to the mathematical beliefs of primary in-service teachers. Only two of these studies were conducted in Australia. Considering this represents a five-year research period, it is fair to say there are few published qualitative studies of in-service primary teachers that directly observe and analyse the relationships between teachers' beliefs and MCK and their effects on classroom practice. A research focus on this aspect of a teacher's personal domain like that proposed in this study is needed to create a more complete picture of critical influences that may affect

classroom practice in mathematics.

Belief is a term frequently used, thereby lending itself to common assumption and methodological difficulty (Thompson, 1992). Poor conceptualisations were proposed as one of the early difficulties in studying teachers' beliefs (Pajares, 1992) and a possible reason for observations of inconsistency in teachers' professed beliefs and instructional practices (Raymond, 1997). Nespor (1987) defined beliefs by their structure and in relation to their strong affective and evaluative components (compared to knowledge). He described beliefs as "loosely-bounded systems with highly variable and uncertain linkages to events, situations, and knowledge systems" (p. 321). Thompson (1992) enveloped the term belief in the term *conception*, using this to describe a teacher's "conscious or subconscious beliefs, concepts, meanings, rules, mental images, and preferences concerning the discipline of mathematics" (p. 132). Philipp (2007) described beliefs as "psychologically held understandings, premises, or propositions about the world that are thought to be true" (p. 259). Beliefs in such systems can be organised in clusters around a particular idea; they may be primary or derivative, central or peripheral (Green, 1971). This categorisation is dependent on their intensity, contextual relevance, and/or relationship to other beliefs.

Attitudes also form part of the affective domain in which beliefs reside. Attitudes can be considered “manners of acting, feeling or thinking that shows ones disposition or opinion” (Philipp, 2007, p. 259). They are considered to change more slowly than emotions but more quickly than beliefs. Emotions are considered less cognitive than attitudes. They tend to change more quickly and are felt more intensely than either attitudes or beliefs. Values are a deeply held belief about something’s worth. They differ from beliefs in their association with a desirable/undesirable dichotomy rather than a true/false one. As they are held more deeply than beliefs, they are less context dependent (Phillip, 2007).

While current literature reflects the ongoing struggle to define the term beliefs, this difficulty also acknowledges its complexity and multifaceted nature and in turn the purpose of using a range of different methodologies to investigate it. Beliefs cannot be observed; they must be reported or inferred (Pajares, 1992). This adds further methodological and analytical complexity as responses from teachers may be influenced by the image they want to project or think they should project, rather than what they actually do or believe. Research investigating teacher beliefs has varied in design depending on the purpose of the study. Most have been qualitative, interpretive, and small scale in nature, although a few studies have attempted large-scale correlation analyses. Quantitative methods are often used to identify the possible structure (nature, orientation) of teachers’ beliefs and investigate relationships among these dimensions.

Patterns associating such beliefs with teaching and learning practices are sometimes sought.

These quantitative methods often include Likert scales, questionnaires, rating scales, and checklists (see, e.g., Gresham, 2018; Nurlu, 2015).

Qualitative methods such as interviews (semistructured and stimulated recall), classroom observations and/or video, hypothetical situation results, and concept mapping are often used after questionnaires and rating scales to delve more deeply into teachers’ self-reported beliefs (see, e.g., Anderson et al., 2005). Some studies have also observed teachers’ practice to support connections made between teachers’ reported beliefs and teaching practices (Skott, 2013).

Using a range of methods, or employing mixed methodology approaches, supports the notion that research around complex behavioural constructs like beliefs is problematic if multiple opportunities to demonstrate the nature of their presence and appearance are not provided.

2.3.1 Nature of beliefs and attitudes

That mathematics teachers are products of the system in which they then function has stimulated investigation into the way teachers think about and understand their role in the classroom. As well as describing the categories and fundamental qualities of beliefs and the dispositions of teachers holding such beliefs, the research literature has addressed the origins of teachers' beliefs—*how* teachers came to construct them—and their structural arrangement, how they *hold* them. In general, the findings of recent research into the nature and effect of teachers' beliefs and attitudes are consistent with previously developed understanding developed from early theorists (Anderson et al., 2005; Harper & Daane, 1998; Nespor, 1987; Stoehr, 2017) with some interesting developments in thought about the mediating role of personal and contextual factors (Bobis, Way, Anderson, & Martin, 2015; Sawyer, 2018).

Nespor's (1987) field-based research on teacher thinking, the Teacher Beliefs Study followed eight high school teachers across a semester, videorecording their classes and conducting semistructured and stimulated recall interviews to generate data. Two of these teachers were mathematics teachers. Results were analysed through a theoretical framework outlining six structural features of beliefs, including existential presumption, alternativity, affective and evaluative loading, episodic structure, nonconsensuality, and unboundedness. Alternativity relates to the conceptualisation of ideal situations that differ significantly from a person's reality. Teachers with such beliefs envisage and attempt to establish classroom structures and formats for which they have no experience beyond an alternative ideal. Such beliefs override concerns and shortcomings in other parts of a teacher's practice as these aspects are justified in the pursuit of the dominant alternative. Episodic beliefs derive their subjective power from particular episodes that continue to frame how teachers understand events later in

time. Both such beliefs serve to define a teacher's goals in the classroom. For example, English teacher Ms. Skylark's ideal of a friendly and fun classroom was a utopian alternative to that in which she grew up (Nespor, 1987). This belief in alternativity was more important than expecting students to finish assignments or covering lessons previously planned. It was concurrently driven by episodic storage of vividly remembered contrasting experiences as a student herself. Nespor suggested that critical episodes such as these, as well as influential and inspiring experiences, could serve as templates for a teacher's own practice.

According to Ernest (1989), a teacher's belief system about the nature of mathematics forms the basis of their *philosophy* of mathematics. Whether consciously or unconsciously held, such philosophies influence what and how they teach. Ernest described three types of beliefs about the nature of mathematics: an instrumentalist view, Platonist view, and problem-solving view. Teachers with an instrumentalist view see mathematics as a set of unrelated facts, rules, and skills to be used towards some external end. Those with a Platonist view see mathematics as a unified and static body of knowledge that is discovered, not created. Unlike instrumentalists, Platonists recognise the connections, logic, and meaning behind the structures of mathematics. Teachers with a problem-solving view of mathematics acknowledge its dynamic and ever-expanding nature. They see mathematics as a process of enquiry open for discussion and revision. Considering the complexity of belief systems, it is possible for a teacher to include aspects of more than one of the above views in their mathematical philosophy and practice, even though such views seem in conflict (Thompson, 1992).

The beliefs and feelings students learn and carry away *about* mathematics are at least as important as the knowledge they learn *of* mathematics (Philipp, 2007). For many educated people, “mathematics is still seen as a discipline characterised by accurate results and infallible procedures” (Thompson, 1992, p. 127), whose basic elements are operations, theorems, and procedures that have little or no connection to their everyday lives. Early informal surveillance as a student forms part of a teacher’s “apprenticeship of observation” (Lortie, 1975, p. 61), information about what mathematics is, how effective teachers teach, and how students should behave. Current research in relation to teachers’ anxiety about mathematics and teaching mathematics continues to support early research connecting such debilitating affective states to teachers’ experiences as students (Gresham, 2018; Stoehr, 2017). Thirty years ago, Harper and

Daane (1997) found mathematics anxiety of preservice teachers was caused by an emphasis on

right/wrong answers, a fear of making mistakes, time constraints when problem-solving, the

feeling they were not smart enough to do mathematics, and lack of confidence in their

mathematics ability. Teacher instruction and attitude during lessons has also been found to

cause anxiety. Preservice teachers have reported feeling embarrassed, humiliated, and made to

feel stupid in front of their peers (Stoehr & Carter, 2012).

Investigating the personal histories of three female preservice primary teachers over an

18-month period, Stoehr (2017) described how these teachers interpreted anxiety as specific

fears related to "loss." These included loss of social belonging, loss of personal identity, and/or

loss of practical competency. All three teachers reported their earliest experiences with

mathematics anxiety related to the point at which ability grouping structures were implemented.

As a child, one of the teachers invented a protective wall, whereby she labelled herself "not a

maths person" (p. 81). The "pervasive and deep seated" (p. 82) nature of the mathematics

anxiety experienced by these teachers was brought little or no relief in adulthood. Stoehr (2017)

suggested such situations may provide partial explanation for resistance to new visions and

methods of teaching mathematics: "They may simply be too afraid to experiment and too resigned to failure" (p. 82). Relatively little is known about mathematics anxiety experienced by in-service teachers and how this affects them over time. There is an indication that teachers with such anxiety are more likely to use traditional teaching methods and concentrate on basic skills, seatwork, and whole-class instruction (Finlayson, 2014).

Research published this year monitored the anxiety levels of 10 preservice primary teachers in the US after their first five years of classroom teaching (Gresham, 2018). These teachers had been assessed as having high mathematics anxiety as undergraduates even after completing a specific content and methods course designed to address this. A quantitative Mathematics Anxiety Rating Scale developed by Richardson and Suinn (1972) was used alongside informal observations, interviews, discussions, and questionnaire-guided narratives.

All 10 teachers involved in this research reported that their mathematics anxiety was "consistently evident" (Gresham, 2018, p. 95) in their mathematics classroom over the five-year period, although most tried to hide it from their students and peers. The decrease in anxiety levels after five years of teaching mathematics was described as minimal. The teachers all said they needed professional learning to increase their SMK and counter what they believed to be their own mathematics curriculum deficiencies. Four of the teachers (all of whom had Master of Education degrees) felt this would be a "career long" (p. 95) focus. It was unclear how much professional learning had been provided by each teacher's school and if this differentiated their experiences. A lack of support to manage and express their mathematics anxiety publicly was offered as one of the explanations for its perpetuity. Because of the effect mathematics anxiety has on a teacher's capacity to learn, Gresham recommended teachers "discard false beliefs and

resist intimidation due to lack of confidence" (p. 105). What is clear from the qualitative findings of this research, however, is how persistent beliefs formed early can be and how difficult they may be to discard.

Understanding teaching from a teacher's perspective is a change in the research agenda that has allowed a more detailed description of the nature of certain teaching behaviours. Part of this analysis has included insight into the beliefs and attitudes mathematics teachers hold and how these influence their practice. Researchers in Australia have called for more attention to the beliefs about the nature of mathematics that teachers have constructed as a result of their schooling and the contributions this makes to the apparent inconsistencies of instabilities in early career (in-service) teachers' practice (Beswick, 2011).

2.3.2 Teacher beliefs and classroom practice

An increasing body of research supports the connection between a teacher's beliefs and attitudes about, and of, mathematics and their classroom practice. Observations have been made about the relationships between beliefs and practice (Stipek et al. 2001) as well as inconsistencies (Raymond, 1997). Researchers have warned about the need to look closely at findings that suggest teachers' beliefs and classroom practices are inconsistent. Classroom teaching situations may have been driven by *leading* beliefs that prevail on other beliefs as the teacher manages their practice (Furinghetti & Morselli, 2016) or be explained by the competing priorities of teachers (Skott, 2001). Researchers have advocated the notion of beliefs as "sensible systems" (Leatham, 2006, p. 92) that are intelligible and purposeful to the holder. Recent studies have used this as a template to dispel assumptions that a teacher's beliefs and classroom practices are misaligned, focusing on the perceptions of the researcher and their assumptions about belief/practice alignment (Francis, 2015).

Ernest (1989) distinguished between espoused and enacted beliefs, arguing that an individual's beliefs about an object, that is, teaching mathematics, are likely to influence how that object is addressed. Sfard (2008) also objectified beliefs, describing them as reified social experiences that independently take on a life of their own. The perceived determinism of such models of beliefs has been challenged. This includes contesting the assumption that teachers'

beliefs are the main obstacle to change and that research on beliefs could remedy problems

related to implementation (Skott, 2013). Methodological and conceptual difficulties

surrounding definition and ways to access teachers' beliefs are cited in this dispute.

Skott (2013) described investigations of beliefs-practice research as a "conceptual-

methodological" (p. 548) impasse caught in a circular argument over which affects the other

and how these data are accurately generated. He argued the situatedness of beliefs (those held

in the classroom differ from those in the research interview) and proposed analysis via patterns

of participation (PoP). Immersed in the theory of symbolic interactionism (Blumer, 1969), PoP

is a processual framework that focuses on the roles of the teacher in action, more specifically

"the teacher-in-multiple-practices-and-figured-worlds as they relate to classroom interaction"

(p. 552). Associated methodologies include the use of multiple open interviews and stimulated

recall in combination with classroom and staffroom observations. Such research leans towards

research on identity as recurrent patterns in teachers' contributions to classroom interaction are

linked to teachers' shifting stories of themselves as professionals.

A PoP approach was adopted in research analysing the relationship between problem-

solving related beliefs, competence, and classroom practice of three Cypriot primary teachers (Andrews & Xenofontos, 2014). Researchers acknowledged in their methodology the "deeply contextual" (p. 303) nature of teachers' problem-solving beliefs, particularly when immersed in cultures that create mental models of teaching that inform how teachers enact their beliefs (Andrews, 2011). Such cultural variables may be hidden, hence the need for research methods that assist in their revelation. A multiple case study approach was adopted to enable construction of detailed narratives that generated data about the complex relationship between how teachers think and act.

Beliefs help teachers make sense of the complex environments in which they work and the "ill-defined and deeply entangled" (Nespor, 1987, p. 324) problems they deal with each day. They act as filters through which teachers assign meaning to their experiences. Ernest (1989) related his conceptions of mathematics to teaching roles, actions, and classroom activities. Teachers with an instrumentalist view are likely to see their role as instructor, emphasising skill mastery and strict following of a text. Teachers with a Platonist view are likely to view their role as explainer, with student learning occurring through reception of knowledge. Such teachers would modify textbook approaches and enrich their curriculum with additional problems and activities. Those teachers with a problem-solving view see themselves as facilitators. They are confident in their ability to pose and solve problems and to present activities that promote learning as the active construction of understanding.

The strength of one's beliefs can be overshadowed by factors like class priorities (building confidence, class management), a perception of an individual class' needs (ages, abilities, ethnicity), as well as general beliefs about children, society, and education (socioeconomic background and status) (Philipp, 2007; Skott, 2001). Such factors may also include the wider affordances and constraints of the school itself as "embedded in this context are the values,

beliefs and expectations of students, parents, fellow teachers, and administrators; the adopted curriculum; the assessment practices; and the values and philosophical leanings of the education system at large" (Thompson, 1992, p. 138).

Skott (2001) introduced the term school mathematics images (SMI) to describe "teachers' idiosyncratic priorities in relation to mathematics, mathematics as a school subject and the teaching and learning of mathematics in schools" (p. 6). This research addressed how novice teachers' unique personal interpretations, or SMIs, relate to the way they deal with the complexity of the classroom. Skott interviewed and observed four novice primary to lower secondary teachers in Denmark who specialised in mathematics. All presented SMIs inspired by the current reform efforts at the time. The case of Christopher was used to highlight the

relationship between SMIs and critical incidents of practice. Christopher viewed his role in the

classroom as the initiator and supporter of investigative activities, allowing his students to take

on responsibility for their own learning. This was evident in one of the observed lessons, but

not another. When confronted with this perceived discrepancy in SMI and practice, Christopher

explained his actions through the lens of competing priorities (managing the classroom and

catering for the confidence levels of students). Skott concluded that Christopher's beliefs had

not changed with the situation but with his goals for the lesson. Ernest (1989) referred to this

higher level of consciousness of a teacher's own beliefs and the benefits for their practice.

Adopting a problem-solving view of teaching, he said, requires reflection on the roles of the

teacher and the learner, on the suitability of the context and models presented, and on ways to

narrow any gap that presents between their own beliefs and lesson goals.

Using a different focus, Anderson et al. (2005) examined the relationship between

primary teachers' problem-solving beliefs and practices through survey, interviews, and

observations. The survey differed from that administered in other research on beliefs. It was

designed to respond to previous criticism about a teacher's ability to recall classroom events

and therefore reflect on beliefs and practices accurately. The survey referred to the use of particular types of problems in mathematics lessons and to examples provided to illustrate the meaning of terms. From the rich data set of interviews of nine teachers, two major factors seemed to impact beliefs and practices: their early experiences as learners and the social context of teaching. Anderson et al. cited the case of Year 6 teacher Gaye who presented with mixed beliefs in the survey and inconsistencies in her views on the place of problem-solving when teaching. Gaye had sound knowledge of contemporary approaches to teaching mathematics and believed they were worthwhile. But her belief in their effectiveness was not necessarily reflected in her consistent use of this approach, in particular when teaching lower ability

students. Such inconsistency further suggested the situated nature of teachers' beliefs—that different sets of beliefs are applied depending on the class or context.

Better understanding of the nature of teachers' beliefs about mathematics teaching and learning and their influences on teacher practice must be examined if we want to cultivate certain mathematical beliefs (Wilkins, 2008). Considering the scope and level of affect referred to in the research addressed in this section, investigation of teachers' beliefs and attitudes in relation to their practice is relevant and important for this research.

2.3.3 Interaction of teacher knowledge and teacher beliefs in classroom

practice

This literature review has presented research that acknowledges the important role teachers' knowledge (Ball et al., 2008) and beliefs (Andrews & Xenofontos, 2014; Skott, 2001) about mathematics and teaching mathematics play in shaping their classroom practice. However, focusing on knowledge or beliefs, rather than their interaction, results in an "incomplete picture" (Thompson, 1992, p. 131). While separate studies have focused on the influence of teacher knowledge and beliefs about mathematics and teaching mathematics on classroom practice, and those who have included these variables in parallel, there is little current research that works at its intersection. This could be related to the complexities of definition. A stable core of beliefs may be extended in unpredictable ways that are meaningful to the holder but not necessarily to others; in this way, beliefs may be simultaneously held as knowledge (Nespor, 1987). Such subjectivity is present in Anderson et al. (2005) who subsumed beliefs as part of the greater term knowledge. This definition included both objective (early mathematics learning, curriculum and resources, preservice, in-service, and postgraduate education) and subjective knowledge (beliefs). It reflected the possible influence of contextual factors from the classroom and school experiences on teacher beliefs and therefore practice. Knowledge held by teachers has been described as a "mental net" (Goldin, Roskin, & Torner, 2011, p. 1) dominated by beliefs. If there is no grounded knowledge available, then beliefs serve as the connections to

maintain the integrity of this net. This view suggests knowledge structures are primarily belief

structures.

For methodological clarity, this study chose to separate the terms knowledge and beliefs.

Knowledge is viewed in terms of the MKT model (Ball et al., 2008) and beliefs as

“psychologically held understandings, premises, or propositions about the world that are

thought to be true” (Philipp, 2007, p. 259).

A search for studies that examined the interaction of in-service teachers’ knowledge and

beliefs published in the last five years was limited; most generated data about teachers’

knowledge and beliefs in parallel. The closest study found to this search focus was

Charalambous (2015) who investigated the intersection of preservice teachers’ mathematical

knowledge, beliefs, and teaching practice. The focus of this multiple case study were three

teachers involved in 13 three-hour content courses and the same amount of methods courses

focused on numeracy. Their content knowledge was assessed through a 41-item assessment

based on components of Ball et al.’s (2008) MKT model. They also completed a survey to

assess how their beliefs aligned with the intentions of the standards-based curriculum of the

US. As the teachers were not yet in their classrooms, teaching simulations were shown and

participants’ reflections noted to further capture their beliefs. Later, the teachers were asked to

create lessons to introduce fractions concepts from two textbook pages. Charalambous' cross-

case analysis suggested that limitations in knowledge and/or beliefs (based on those that align

with standards-based curricula) could not compensate for the other as each made distinct

contributions to the teachers' decisions and actions. Such research suggests the interaction of

knowledge and beliefs and practice is dynamic and complex, warranting further investigation

in other contexts under different pressures and contextual limitations.

2.3.4 Changes in beliefs and attitudes

The structural complexity of beliefs systems creates challenges for those attempting to

change the beliefs and attitudes of individual teachers. Learning is a process of change in

internal mental states. This process can come into conflict with beliefs, which are considered

stable and resistant to change. The affective nature of beliefs, and their relationship to personal

memories, experiences, and assumptions, puts them beyond simple knowledge presentation as

a means of change. Research has suggested that teachers tend to assimilate new ideas to fit

existing schemata rather than replacing or reorganising new ideas as presented (Goldin et al.,

2016; Pajares, 1992). Beliefs are linked to the self-concept and self-efficacy of the teachers.

These function as a self-assertion that protects the teacher from uncomfortable ideas (Goldin et al., 2016). Beliefs that are held deeply and are embedded early are harder to change; they require a “gestalt shift” (Nespor, 1987, p. 321) to make accommodation of new ideas plausible. Teachers dissatisfied with their existing beliefs are more likely to replace them (Lilejdahl, 2010), but even then the new beliefs must be intelligible and plausible before such accommodation can take place (Rokeach, 1986).

The role of reflection is identified as crucial for supporting teachers’ changing beliefs and practices (Philipp, 2007; Stipek, 2001). Questions about how teachers can be encouraged to foreground their own beliefs in the change process have been asked (Beswick, 2014; Francis, 2015), although this assumes that individuals can clearly articulate their beliefs. Focusing

reflection on personal factors outside teaching and teacher education (their parents, spouses, children, and friends) has been suggested as a way to initiate reflection on beliefs (Sawyer, 2018). For teachers to offer their beliefs with the view to be externally challenged requires mutual trust and respect. Prompting may be needed to expose categories of beliefs teachers are unaware of and/or are willing to share (Beswick, 2014).

A teacher’s beliefs support their sense of efficacy (Katz & Stupel, 2016; Nurlu, 2015; Philipp, 2007). Teacher self-efficacy is defined as “the belief in one’s capabilities to organise and execute the courses of action required to manage prospective situations” (Bandura, 1995, p. 2). If teachers believe mathematics is a set of facts and tools and they have mastered these, they are empowered. Belief in teaching-as-telling provides an attainable model for success, especially in relation to mathematical procedures that cannot conventionally be learnt without teacher intervention. Classrooms directed by teachers who believe children should listen and watch to learn provide clear messages about specific roles and are easily evaluated in terms of their success. Such self-affirming behaviours involve individual reflection of some kind (Philipp, 2007). Self-efficacy, then, is linked to professional behaviour, affecting a teacher’s beliefs in their capability, motivation, and success or failure (Nurlu, 2015).

Results from research of primary in-service teachers in Turkey demonstrated differences in characteristics of teachers with high and low self-efficacy (Nurlu, 2015). After completing a self-efficacy questionnaire, four of the 33 teachers in this research were interviewed. Subfactors of the scale used to measure the teachers’ self-efficacy were then used to drive the interview questions. These included efficacy in teaching mathematics, beliefs about motivating students, and effective teaching. Two teachers who were high on these scales and two that were low on these scales were chosen as participants for the interviews. Results indicated that teachers with higher self-efficacy also reported higher levels of effort and persistence with students and were more open to new ideas and methods. These teachers also took more responsibility for students' success and failures, whereas teachers with lower self-efficacy were more likely to see outside factors such as parents as the main reason for such results.

Researchers have proposed two types of beliefs in relation to teacher efficacy (TE): personal teaching efficacy and general teaching efficacy (Katz & Stupel, 2015). Personal teaching efficacy (PTE) involves a feeling of confidence with regard to their own teaching abilities ("I can"); this may be present in one context but not in another. General teaching efficacy (GTE) relates to the potential of teachers to overcome adverse general circumstances presented at school (unmotivated students, unsupportive student home environments). These PTE and GTE constructs are independent; a teacher may believe in one but not necessarily the

other. This kind of separation supports Green's (1971) proposition that beliefs may be clustered (TE = PTE and GTE) but are held separately. This premise was supported in research investigating ways to enhance the self-efficacy beliefs of six midcareer primary teachers in Israel (Katz & Stupel, 2016). These female teachers felt a lack of PTE to change the outcomes of students in their mathematics classes due to difficulties in managing the behaviour of their students and consequential difficulty in focusing on learning experiences. All were observed by researchers in class and post-lesson discussions were conducted. Semistructured interviews were held at the beginning and the end of the research period. During the seven months of the research, these teachers participated in professional learning each week for two hours, involving more than 200 challenging mathematics activities overall. The positive psychological and emotional states promoted by this supportive and ongoing professional learning resulted in increases in PTE (success in completing challenging mathematics tasks) and GTE (beliefs about their ability to overcome the influences of students' home environments). Evidence of improvement in their students' learning when tasks were implemented in the teachers' classrooms also led to higher self-efficacy beliefs.

Whether teachers perceive change as a challenge or a threat is presented as pivotal in Gregoire's (2003) cognitive-affective model of conceptual change (CAMCC). The CAMCC specifies mediators of beliefs change and accords powerful merit to teachers' pre-existing beliefs and teaching experiences. The model suggests that teachers presented with a change situation, like mathematics reform agendas, make a decision about whether these initiatives implicate change for themselves, how stressful this is, how motivated they are to become involved, and whether they have the ability (time, supportive colleagues, SMK) to implement them. Gregoire theorised that strong motivation and sufficient time leads to a challenge appraisal of the situation, whereas weak motivation and insufficient time for reflection leads to an appraisal of threat. A challenge appraisal may lead to true conceptual change or no belief change. A threat appraisal, however, leads to superficial or no belief change. As seen in previous research, teachers with low self-efficacy perceive stressful situations as more threatening.

Examination of the processes of growth and change, and the role that personal and professional domains of a teacher's world have on this change, are important aspects of this research. The current study has the potential to assist the exploration of beliefs and practice of in-service teachers called for by Thompson (1992). Awareness of what beliefs teachers hold and how they hold them may give facilitators of professional learning greater insight into the extent to which existing beliefs can be challenged and the processes by which they may be changed.

2.3.5 Summary of literature on teacher beliefs and attitudes

While the trend of researching teachers' mathematical beliefs has been maintained as an area of interest, there are more studies focused on secondary teachers (Beswick, 2011) and students' beliefs (Dimarkis et al., 2014) than those targeting the effect of beliefs about mathematics and mathematics teaching on primary teacher practice. Reviews of research in mathematics education in Australasia have identified a renewed interest in the anxiety of preservice teachers, but few of these were related to teachers of primary years. Reviewers have called for an increase in the scope of research on affect to investigate how teachers' beliefs about mathematics and mathematics teaching influence their practice (Attard, Ingram, Forgasz, Leder, & Grootenboer, 2016).

2.4 GROWTH AND CHANGE

Literature on the process of educational change presents a move away from mass innovation packages created independently of teachers and the view of teachers as relatively passive adopters and implementers. Lack of success in such a formulaic approach has resulted in a change in perspective, from a focus on the innovations to looking primarily at the perspective of those driving them, including teachers and administrators (Fullan, 2016). This change of agency has reconceptualised the term *teacher change* and therefore analysis associated with it. This section outlines such a progression, including the models used to represent teacher growth and related mediating influences. Research-based principles of effective professional learning and strategies to achieve them are described, including those that relate specifically to the research purpose of this study.

2.4.1 Process of teacher change

There is widespread international acknowledgement that ongoing professional development of teachers is necessary to maintain a professional body with the knowledge and skills to improve student outcomes (AAMT, 2006; NCTM, 2014; Sullivan, 2011, Timperley, 2011). Apparent in the current research literature is the concurrent use of the terms *professional development* and *professional learning*. Individual ERIC searches for research papers using these terms in the past five years resulted in very similar results, with 9,645 papers using the term professional learning and 11,946 using the term professional development. While some researchers have described the difference as semantics (Brock, 2016), others use the term professional development to mean the range of opportunities provided to teachers to promote the process of professional learning (Darling-Hammond, Hyler, & Gardner, 2017; New South

Wales Education Standards Authority, n.d.). A background paper designed to inform Australia's professional development framework for teachers and school leaders referred to both terms and their relationship to the relevant AITSL standards (Timperley, 2011). It referred to "professional learning and development that is inclusive of both formal and informal opportunities for teachers and leaders to deepen professional knowledge and refine professional skills as described in the relevant Standards" (p. 4). The NSW DoE's *Policy of*

Professional Learning (n.d.) released this year also referred to a range of training opportunities. These included online, face-to-face, individual, and shared, which "provide opportunities for professional discourse, interaction, practice, reflection and analysis" (para. 6). This professional learning, it continued, should promote ongoing professional growth and improved student outcomes. Beyond specific terms, the difference in approach to teacher growth and change appears to be conceptual, moving away from one-off prepackaged sessions or change as result of training to a view of learning and development as ongoing—from a product to a process orientation.

These perspectives on the term *teacher change* are incorporated in those identified by Clarke and Hollingsworth (2002). The terms are interrelated, and many overlap. They are

- Change as training—change is something that is done to teachers, i.e., teachers are *changed*
- Change as adaptation—teachers *change* in response to something; they adapt their practices to changed conditions
- Change as personal development—teachers *seek to change* in an attempt to improve their performance or develop additional skills or strategies
- Change as local reform—teachers *change something* for reasons of personal growth
- Change as systemic restructuring—teachers enact the *change policies* of the system
- Change as growth or learning—teachers *change inevitably though professional*

activity; teachers are themselves learners who work in a learning community. (p. 948, emphasis in original)

A change in perspective, however, does not necessarily ensure transference from professional development opportunities to classroom practice (Cole, 2012). Key findings from the 2013 Teaching and Learning International Survey (TALIS) found although Australian teachers reported high participation rates in a range of professional development options (course, conferences, in-service training, networking, and collaborative research), fewer Australian teachers than the TALIS average reported these experiences had a "meaningful impact on their capabilities" (Organisation for Economic Co-operation and Development, 2013,

p. 2). Such disparities posit the need for further research on the process of teacher change, an important factor in the overall success of professional learning initiatives yet one that many fail to consider (Justi & van Driel, 2005).

At a time when "staff development" was the term widely used, Guskey (1986) described the major outcomes of effective staff development as change in classroom practices, student learning outcomes, and teachers' beliefs and attitudes. He suggested the temporal sequence of these outcomes was important to create the best possible conditions for change. His linear model (see Figure 2.4) presented the order of outcomes Guskey (1986) considered most likely to achieve the desired change. Research support for the model generally involved large-scale program implementation efforts. A study analysing the implementation of mastery learning concluded that only teachers who used the mastery learning procedures and saw evidence of change in student learning outcomes positively changed their own beliefs and attitudes in relation to this professional development program and their self-efficacy as teachers (Guskey, 1997). Changes in beliefs and attitudes of teachers, then, were as a result of changes in the learning outcome of students.

Figure 2.4. Guskey's model of the process of teacher change (1986, p. 160).

While acknowledging the model's limitation in accounting for all the variables associated with the teacher change process, Guskey (2002) suggested the "process of teacher change through professional development is complex but not haphazard" (p. 389). Significant change in beliefs and attitudes is unlikely if there is no evidence of "improvements in student learning" (p. 383). Careful attention to the order of change events, therefore, is needed.

Challenging the linear focus of change processes like Guskey's model in their review of literature on teacher learning, Borko and Putnam (1997) concluded, “The order in which beliefs and practices are addressed in staff development programs may not be that important. What is critical is that both practices and beliefs become the object of reflection and scrutiny” (p. 702).

Such scrutiny was the focus for Wood, Cobb, and Yackel (1991) in a yearlong professional development experiment about how children learn in a classroom, which emphasised a constructivist (problem-centred) approach in mathematics. As their research proceeded, it

expanded to include an analysis of situations where the teacher was also learning. Wood et al. (1991) realised that it was not until the teachers started questioning their own practice and the effect it may have on their students did the strong motivation arise to change. The classroom provided a context for “gradual constructions and transformations” that were “characterised by major dilemmas and conflicts that the teacher encountered as she taught” (p. 597). Their summary suggested that the ongoing process of conflict, reflection, and resolution provided opportunities for teachers to learn and for their beliefs to change.

Despite suggestions of such complexity in teacher change, Desimone (2009) also offered a linear model that presents as a causal chain. The order of the elements presented in the model differs from Guskey’s; that is, changes in teachers’ knowledge and beliefs precede change in their practice. The pathways of possible change are argued as nonrecursive and interactive,

suggesting that the order of change in not necessarily fixed. Acknowledged along this model is

the influence of context, including teacher and student characteristics, curriculum, school

leadership, and the policy environment. Like Guskey's (1986) model, the arrows connecting

elements of Desimone's model do not make clear the process that is occurring beyond the causal

suggestion *leads to*. The linear and positivist presentation of Desimone's (2009) model and the

suggestion that it can operate in a nonlinear way do not make its utility clear, inhibiting its

potential for forecasting professional learning outcomes (Boylan, Coldwell, Maxwell, &

Jordan, 2017).

2.4.2 Interconnected model of teacher professional growth

Evidence of multiple and cyclic growth pathways in the process of teacher change was

generated by Clarke and Hollingsworth (2002) and modelled on the IMTPG. This model was

developed as part of two longitudinal investigations of Victorian teachers, the ARTISM and

EMIC studies, and the Negotiation of Meaning project (Clarke & Hollingsworth, 2002; see

Figure 2.5). It supported the interpretation of mechanisms that trigger change (or growth) in

one or more of the four domains of a teacher's world: personal, external, of practice, and of consequence. Such domains incorporate those modelled by Guskey (1986) but moved away from the suggestion of a defined sequence of events leading to change.

Figure 2.5. The interconnected model of teacher professional growth (Clarke & Hollingsworth, 2002, p. 951).

In their research, the IMTPG provided a lens to describe interaction between domains in a particular change environment, the mediating processes of enactment (putting a new idea into action) and reflection, and the resultant (professional) growth or learning. The model aligns with an increasingly popular view of change as professional growth, acknowledging teachers as active and reflective learners socially situated within a learning environment (Clarke & Hollingsworth, 2002). Change may occur in one domain of a teacher's world but not lead to change in other domains. A *change sequence* may occur, represented in the model when two or more of the identified domains are connected by reflective or enactive links. This may be momentary experimentation, not necessarily sustained. A *growth network* represents considered and long-lasting change in cognition and/or behaviour.

Modelling change on the IMPTG is consistent with both situative and cognitive theories of learning. The situative perspective is recognised in the model through flexible representation of an individual teacher's practices as they grow and change within an acknowledged change environment. The cognitive perspective is acknowledged through focus on the construction of different types of knowledge by individual teachers in response to professional development programs and activities within their own classroom. The extent to which the change environment or social setting influences particular practices associated with learning can be represented by the mediating processes of enaction and reflection. These processes usefully connect to practice and cognition (respectively).

Researchers conducting their study in a limited timeframe have reinterpreted the concept of a growth network on the IMTPG, using the criterion of complexity to identify the difference between "superficial change" (Justi & van Driel, 2006, p. 443) and growth. Complexity was defined as the number of relationships between IMPTG domains rather than how long the change was maintained. The more relationships that were represented, the more complex a change was considered to be. It is not clear in the analysis how many relationships had to be represented before it was considered growth.

Beyond the reinterpreted use of the model for data generation over a limited time, this model has been used to analyse impact across a wide range of contexts, including evaluating the coparticipation of leaders and teachers in professional development (Hilton, Hilton, Dole, & Goos, 2015), changes in mathematics teachers' knowledge, beliefs, and attitudes as a result of participation in network meetings (Witterholt, Goedhart, Suhre, & van Struen, 2011), and reciprocal peer coaching (Zwart, Wubbels, Bergen, & Bolhuis, 2007). It has also been used to investigate the relationship between beliefs about teaching science and classroom practice after video-supported collaboration with peers (Lebak, 2015) and as a framework to synthesise literature about professional learning in mathematics (Goldsmith, Doerr, & Lewis, 2014).

Google Scholar citations reported that the paper in which the model had been first proposed had been cited 1,953 times (as of 8 April 2018).

In their analysis of data related to the growth of beginning science teachers' knowledge on models and modelling, Justi and van Driel (2006) used the three suggested functions of the IMTPG (analytical, predictive, and interrogatory) to address specific research questions. As an analytical tool, they represented the growth and change sequences in relation to PCK, evident after participating teachers implemented lessons planned during the research. The predictive function was used to analyse mechanisms within the model's domains that best promoted change. Justi and van Driel (2006) suggested focusing the interrogatory function of the IMTPG on the relationships already established through its analytical function, increasing its explanatory potential in their research. Use of the IMTPG provided these researchers "a framework to monitor and understand the development of [these] knowledge aspects" (p. 448) and their idiosyncratic nature.

Many researchers have used the IMPTG to identify pathways of change as a result of professional learning (Lebak, 2015; Wilkie & Clarke, 2015; Witterholt et al., 2011; Wongsopawiro, Zwart, & van Driel, 2017; Zwart et al., 2007). Most of the studies located for this review involved science and mathematics high school teachers, while one involved an upper primary teacher. All but one of the studies had six or fewer participants, enabling detailed analysis of the factors that mediate change in knowledge, beliefs, and behaviour. The largest participant sample involved growth analysis of 12 high school teachers in a one-year action research project during the Mathematics and Science Partnership (MSP) program (Wongsopawiro et al., 2017). Teachers in the MSP attended a two-week summer institute presented by academic staff on science and mathematics topics, were supported in conducting action research in their classrooms, and were provided with four follow-up sessions. Teachers' reflections on progress were self-reported through an electronic journal then addressed at follow-up workshops. An interview after the end of the research project probed more deeply into teachers' journal responses. These data allowed the researchers to generate 48 IMPTG pictograms modelling teacher change, identifying pathways with common entry points, sequences of change, and end points.

Findings in the MSP research identified distinct pathways that led to changes in PCK, including those that involved the domain of consequence (DC) and those that did not. These distinguished between *complex* growth and *simple* growth (respectively): in other words, those who reflected on their students' learning in the process of change and those who did not. Similar pathways of PCK development were also evident in the data, allowing Wongsopawiro et al. (2017) to compare two distinct groups of teachers: those who consistently reflected on their actions and changes and those with fewer reflection pathways. In the first group of teachers, focused and structured reflection on students' learning was an important catalyst for PCK development. The powerful influence of the external domain (ED) on teachers' PCK was also acknowledged, including the deep expertise of university staff on participants' knowledge of science curricula and knowledge of student understanding. In the light of these findings, Wongsopawiro et al. called for greater attention on how the DC interacts with other domains and the important role that expertise plays in assisting teachers to construct new knowledge.

Models of professional learning developed by Opfer and Pedder (2011) and Evans (2014) offer different conceptualisations, the former a systems model and the latter a cognitive learning model that theorises about the individual microlevel processes. The Opfer and Pedder model is based on synthesis of research findings in professional development literature. The teacher, school, and the learning activity system are offered as key elements. Unlike Guskey (1986), Desimone (2009), and Clarke and Hollingsworth (2002), this complexity model is not diagrammatic; rather, it is proposed as a set of nested systems that interact to influence professional learning. Relationships include orientations between and to systems within this nest. Like Clarke and Hollingsworth (2002), it offers the potential to propose causal chains and pathways of teacher learning, and takes into account both formal and informal learning.

Omitted

in this model is the relationship between student outcomes and teacher learning and the suggested influence of wider systemic influences (like national assessments and implementation of system-wide curriculum).

Evans (2014) model takes a cognitive perspective; the situatedness of professional learning in a social context is not included. The premise of the model is that professional development occurs when a *better way* of doing something is recognised by the individual teacher. This is reflected in the model's focus on three components of individual learning: behavioural, attitudinal, and intellectual. Each of these components is divided into dimensions of change; for example, behavioural development is broken down into the elements of processual change, procedural change, productive change, and competential change. A teacher recognises a better way of doing something, and then change occurs in one or more of these dimensions, possibly across the three major components. This model accounts for formal and informal learning, like those mentioned previously. Evans argued overlaps between her model and the IMPTG in relation to "micro-level development" (2014, p. 864) and Clarke and Hollingsworth's (2002) description of *change sequences.* She challenged, however, the adequacy of enaction and reflection in fully representing a teacher's mental internalisation.

Evident in studies like those highlighted in this section is the pivotal influence of reflection as a catalyst for change. While there is wide recognition about its importance, differences in definitions and the most effective strategies for promoting reflection that effect

change have been raised.

2.4.3 Reflection and the process of growth and change

Reflection is an important internal process when studying the process of teacher change (Darling-Hammond & Bransford, 2005; Sowder, 2007), but a wide range of definitions of the term prevents consensus on its impact. Influential theorist Dewey (1910) defined reflection as the “active persistent and careful consideration of any belief or supposed form of knowledge in light of the grounds that support it, and further conclusions to which it contends” (p. 6). Such serious consideration involves “a state of perplexity, hesitation, doubt” (p. 9) and a need to suspend judgment during the ensuing inquiry, a situation that is likely to be “somewhat painful” (p. 13). Schon (1983) drew greater attention to the capacity of professionals like teachers to reflect while in the act of teaching. While acknowledging the influence of Dewey’s ideas on his own, Schon (1992) differentiated his theory of inquiry, proposing “reflective practice” (p. 123) as his version of Dewey’s “reflective thought.” Schon’s reflection-in-action suggests that

divisions between thinking and doing are artificially imposed “because experimenting is a kind of action, implementation is built into . . . inquiry” (1983, p. 68).

Mewborn (1999) noted the lack of consensus about the term reflection but identified three connecting conceptions in the literature: that reflection is qualitatively different to recollection or rationalisation, that action is an integral part of the reflective process, and that reflection is both an individual and shared experience requiring some level of prompting or probing. Opportunities may be provided for teachers to reflect, but this does not necessarily mean that reflection will happen (Knight, 2002). Reflection, unlike recollection and rationalisation, is not a “natural state of mind, nor is it easy to make it a habit” (Mewborn, 1999, p. 317), perhaps because of the associated confusion and discomfort. Researchers have raised the need for reflection to be accompanied by “some form of confrontation” (Day, 1993, p. 83) to promote substantive growth and avoid “closed-circle thinking where the essential rightness of existing thought and action is habitually confirmed or only mildly troubled” (Knight, 2002, p. 233).

A number of researchers have used video-stimulated recall followed by collaborative discussion to assist the reflection process. Acknowledging the role of reflection in questioning and modifying teacher beliefs, Senger (1999) used beliefs questionnaires and statement cards in combination with reflection on videorecorded practice to probe alignment issues of three primary teachers implementing new reform curriculum. This video-and-theory reflection incorporated the practical argument approach outlined by Fenstermacher (1994) and Richardson (1994) to help teachers deliberate about and evaluate their thinking and action.

Strategies such as structured rubrics to focus collaborative video analysis have also been used to promote reflection on practice that promotes professional growth (Dostal & Wolbers, 2015).

The importance of an external perspective (e.g., researcher, expert) in supporting self-reflection has also been raised (Durand, Hopf, & Nunnenmacher, 2016; Geiger, Muir, & Lamb, 2015).

Handal (1990) speculated on a hierarchy of reflective practice: Level 1 was action, Level 2 practical and theoretical reasons, and Level 3 ethical justification. His research with teachers in Norway found reflection mostly occurred at the level of action; that is, deciding what to do, when, and how to do it. Their reflections rarely referred to the reasons for such action (Level 2) or justification for them (Level 3). Reflections at these higher levels were not in high demand in the “busyness” of school.

Moving beyond practical reflection to critically studying practice at the levels of reasoning and ethical justification may be dependent on a variety of factors, including teachers’ abilities and skills in these areas, the extent to which they are systematically applied, the frequency of such practice, and the existing psychological and social context (Day, 1993). If in

the confrontation of ideas “teachers will need active, planned and skilled challenge and support” from “skilled helpers . . . critical friends, *trusted* colleagues . . . who have not only the technical abilities but also human relating/interpersonal qualities and skills as well as time, energy and the practice of reflecting on their own practice” (Day, 1993, p. 88, emphasis in original), then the context in which teachers are immersed affects the extent to which growth and change can occur. If the collegiality offered has to extend beyond “comfortable collaboration” (p. 88) for change to occur, the access teachers have to such resources (including personnel and time) is affected by where they work (school and system).

The ideal of teachers as reflective practitioners and lifelong learners is addressed in the literature related to preparing teachers for change (Darling-Hammond & Bransford, 2005). Dewey (1933) identified four attitudes needed to cultivate reflective thinking: open-mindedness, whole heartedness, responsibility (considering the consequences of what was learned), and directness (addressing a problem because it is worth doing). The concept of adaptive expertise builds on these notions, pointing to the willingness of teachers to continually expand their core competencies, ideas, and beliefs despite the emotional consequences of associated questioning and realisation (Hammerness, Darling-Hammond, & Bransford, 2005). Such restructuring may affect the teachers’ efficiency in the short term (on a novice to expert scale) but the process is seen as worthwhile because of the increased ability to apply innovative ideas in the long term. Preparedness for change, then, also involves acceptance of the necessary state of some discomfort and the view that confusion has its virtues (Brown, 1993). Also described as dissonance (Delaney, 2015) or productive friction (Bakkenes, Vermunt, & Wubbels, 2010; Ward, Nolena, & Horn, 2011), entering and reconciling this state of unease is posited as essential for deeper change in teachers' work to occur.

Frykholm's (2004) case study research of long-term implementation of MiC curriculum with eight U.S. middle school teachers used a framework that categorised discomfort domains as cognitive, belief driven, pedagogical, and emotional. To make sense of the various manifestations of discomfort present, he proposed the notions of debilitating and educative discomfort. Debilitating discomfort emerged when teachers did not have the content knowledge to engage in the mathematics, a situation made more severe when their beliefs ran counter to those presented in the curriculum. Educative discomfort on the other hand is an instructive rather than negative state and part of a process that leads to growth. Teachers with a tolerance for educative discomfort may mirror their own learning state, leading their own students towards discomfort to promote deep learning. Frykholm linked the idea of discomfort to self-efficacy research (Bandura, 1997; Pajares, 1996), in particular how much effort teachers will invest in a challenging activity. The strength of this self-efficacy filter influences whether the discomfort becomes more debilitative or educative and therefore the teacher's response to new challenges like the MiC curriculum.

2.4.4 Time and context

Most researchers of teacher professional learning suggest growth and change takes time (Darling-Hammond, Wei, Andree, Richardson, & Orphanos, 2009; Kutaka et al., 2018). Change requires risk of failure that is counterintuitive to the embedded psyche and resultant practices of most teachers; commitment to student growth is strong and the risk of students learning less than current practice offers is a demanding situation (Guskey, 2002). Some researchers have suggested that professional development projects should be at least one year in length (Borko & Putnam, 1997; Sowder, 2007), with most large-scale projects proposing longer periods of time (Clarke et al. (2012), especially if teachers' belief systems are being challenged. For such gradual transformations to occur, time is needed to resolve some of the major personal dilemmas for the teachers involved, including changes in the social norms of the classroom and the teacher's negotiated role in these.

Countering this argument of profound change taking time, Lilejdahl (2010) cited more than 40 observed cases of "significant and robust changes" (p. 411) in primary, middle, and

high school mathematics teacher practice in short periods of time. Five distinct mechanisms

were presented in relation to the teacher change: conceptual change, accommodating outliers,

reification, leading belief change, and push–pull rhythm of change. Within the examples given

were changes in beliefs and practice in teachers within a two-month period. This example

involved an experienced primary teacher who had enrolled in an elementary mathematics

education master’s program with the view to improving her practice, in particular moving away

from more traditional teaching practices. This teacher had rejected her own practices and was

searching for a new paradigm; the “belief rejection followed by belief replacement” (p. 414)

that followed may have been strongly facilitated by this desire to change, rather than an imposed

external source insisting on change.

The intensity of a professional learning program has been referred to as both the number

of hours involved in its implementation as well as the length of time over which these are

distributed (Kennedy, 2016). It may involve the volume of information transmitted in this time.

Such intensity appears to be more effective in promoting teacher learning when the program

design is aimed at developing teachers’ insights into enacting new ideas into their own systems of practice rather than those that are heavily prescriptive and allow little flexibility or personal judgment.

The extent to which career stage affects the process of growth and change has been investigated and potential career cycle trajectories suggested (Drake, 2002; Steffy & Wolfe, 2001). Studying high school teachers in Switzerland and the self-nominated stages or phases of their own careers resulted in categories of career trajectory that offered further investigative potential in relation to items like school improvement (Huberman, 1998). In the interview schedule, teachers were asked to what extent they felt well-designed innovations were worth the effort and complications involved at different points in their career. Trends in these data suggested teachers were less willing to invest heavily in innovations as their career progressed. Teachers in the mid to late phases of their teaching careers were identified as having “positive focusing,” “negative focusing,” and “disenchantment” phases after implemented reform (p. 126). While the attitude of the first is more optimistic than the other two, each of these phases suggested a contracting of commitment and energy to external stimuli, including whole-school innovations. While acknowledging difficulty in presenting modal trends as present in all teachers, Huberman raised the possible difficulty of changing school practices significantly where a number of teachers are at different points in their career cycle.

The hypothesis that career life cycles and professional development needs are connected

was investigated in a project involving the trialling of reform curriculum in California (Drake,

2002). The researchers used career stage as a framework for identifying patterns in teachers'

responses to the reform curricula. Career stage was defined in terms of formal years of teaching,

that is, 1 to 3, 4 to 6, 7 to 11, 12 to 20, 21 to 30, and more than 31 years. Results from the initial

large-scale survey of nearly 600 teachers in the curriculum trial found a high correlation

between career stage and measure of life stage (younger teachers had less experience and older

teachers had the most experience). The subsequent qualitative phase of the study included six

female teachers: two early career, two transitioning from early to midcareer, and two midcareer.

These teachers were observed teaching mathematics from five and 15 times over a school year.

All were interviewed about their mathematics life stories and asked to recall several key points

in their experiences of learning and teaching mathematics. While all teachers were open to some

degree to reform, Drake found the teachers transitioning from early to midcareer were most

explicitly reflective about their process of change in this reform and most open to making large-

scale changes in their practice. Early career teachers needed more practical supports to help

them implement the reform, and later career teachers needed to be convinced of the overall

coherence of the reform's instructional benefit. Drake's consideration of career stage as an

indicator of development within a particular context (in schools and in the classroom) acknowledged the influence of time in such a specialised context. Despite the recognised limitations of these data, the researchers proposed a link between teachers' career development and their patterns of reform interpretation and implementation.

The context of teachers' learning is raised in both of these studies. In the case of Huberman (1998), the wider context of major structural reform of schools in the time period preceding the research (1982–1985) is mentioned, and in the case of Drake (2002), the ongoing education mathematics reforms present in California at the time were relevant. How these innovations and other reform initiatives were managed, and the effect they had on the teachers' response to change, are also considerations.

2.4.5 Preparedness for change

The efficacy of using career stages as indications of response to professional learning is complicated by studies that categorise teacher attitude and preparedness for change. Doyle and Ponder (1977) described most teachers as "pragmatic sceptics" (p. 3) who (tentatively) consider the practical application of a change proposal before them, foregrounding their own decision-making and rejecting externally imposed mechanisms that hold little classroom relevance. This practicality ethic is conceptualised as instrumentality (classroom applicability), congruence (with current practice, contextual fit, and current beliefs), and cost (ratio of effort vs effect). It is posited as a “useful interpretative tool for understanding how teachers make decisions” (p. 9). As part of the UC Assessment Project (Borko & Putnam, 1997) involving professional development of Grade 3 teachers, researchers also observed teachers as “cautious—even sceptical—about educational innovations that placed additional demands on their time and energy” (p. 236). Teachers in their study were willing to try out new ideas but wanted to be convinced of their worth. This caution was not restricted to teachers of a particular career stage.

A quasi-developmental pathway of caution was proposed in the Concerns-Based Adoption Model (CBAM) (Fuller, 1969). Based on Fuller’s work, the CBAM proposed that teachers’ concerns move from self-concerns (awareness, informational, personal) to task concerns (management) and finally impact concerns (consequence, collaboration, refocusing). This pathway can be influenced by factors like how innovations are implemented (timeline, support structures offered, leadership) and the experience of the teachers involved (Hall & Hord, 2015). Teachers can experience more than one stage of concern (SoC) at the same time, with different degrees of intensity. This literature suggests that context, personal needs, and individual development are important considerations in the process of change, some of which may be related to but not restricted by career stage and experience.

The influence of context is addressed in literature concerning the social and distributive nature of schools and therefore teacher learning. Lave and Wenger (1991) rejected the notion that learning was either completely subjective or fully dependent on social interaction. They proposed a decentred view of where and how learning occurred, raising the concept of learning as “legitimate peripheral participation in ongoing social practice” (p. 64). In this view of learning, knowledge and skills are developed and changes in identity occur as individuals align themselves with the community’s practitioners. Learning as situated social practice puts a focus on the person as the “person-in-the-world” and member of a sociocultural community, promoting a view of knowing as “activity by specific people in specific circumstances” (Lave & Wenger, 1991, p. 52). Teaching is a specific profession in a unique context involving interaction among teachers, and teachers and students, in the learning process. Advantages and limitations of this situation have been addressed by research investigating collaborative work, including learning opportunities situated in teachers’ own classrooms (Borko & Putnam, 1997; Hill & Ball, 2004; Lebak, 2015; Wood et al., 1991). This literature suggested professional conversations support teachers to make sense of new practices and provide structures that promote encouragement and support to persist through difficulty. The positive contribution of support from researchers and outside expertise in the change process is also addressed, particularly in relation to clarification of ideas and provision of supportive resources to sustain emerging practices (Geiger et al., 2015; McGee, Polly, & Wang, 2013; Wolf, 2010).

Context can mediate the “strong utopian ideal” (Hill, 2009, p. 474) of teachers as continual learners striving for improvement. Like other professionals who are part of a complex and integrated system, teachers respond to the current incentives, norms, and models in which they are immersed. Opportunity for change, then, may not be about willingness or attitude but the quality of experiences offered and constraints imposed on accessing these. Collaboration and coaching situations that are mandated by schools can be counterproductive, turning “genuine teacher inquiry into contrived rituals of enforced collegiality that actually make teachers inclined to collaborate less” (Hargreaves, 2010, p. 290).

2.4.6 Features of effective professional development

An increased commitment to, and recognition of, the need for quality professional development has driven the need to identify characteristics of approaches that successfully foster teacher change (Borko & Putnam, 1997). Comparison of research-based principles to guide the design of effective professional development demonstrates clear overlaps, including the recognition of the importance of a theoretical base, a clear vision regarding goals for the professional development, the pivotal role of support to take instructional risks, the benefits of a sustained project focus, and the need for meaningful feedback (Clarke, 1994; Guskey, 2002; Hawley & Valli, 2000, Timperley, 2011).

In reviewing the research around the mathematical education and development of teachers, Sowder (2007) identified six (interrelated) goals of professional development in mathematics. These are the development of a shared vision, MCK, an understanding of how students think about and learn mathematics, PCK, an understanding of the role of equity in school mathematics, and a sense of self as a teacher of mathematics. These components recognise that mathematics teacher change is a gradual, lengthy, and potentially uncomfortable process. They suggest movement away from providing isolated professional development experiences and a move towards focusing on techniques that build teachers' capacities, including the strengthening of content knowledge.

Ten key principles on teacher professional learning assimilated by Timperley (2008) focused particularly on research demonstrated to have a positive effect on student outcomes. Four of the first 10 principles relate to an emphasis on professional learning opportunities that improve teacher content and PCK. References are also made to providing multiple opportunities to learn and apply new information in context. It was suggested that collegial interaction is more effective in promoting professional learning if it is “focused on becoming responsive to students,” avoiding the common pitfall of the “norms of politeness and the absence of challenge” (p. 19). The presence of external expertise is recommended to challenge assumptions and present new possibilities.

Expertise to support the change process is also promoted in literature that calls for differentiated approaches that stimulate individualisation and ownership (Clarke, 1997; Tomlinson, 2005). There is, however, little discussion in the professional learning literature about the expertise of those designing and delivering professional learning. Effective programs and opportunities are often offered on smaller scales by providers and individuals who have worked with many teachers, are familiar with the challenges they face, and use their own experiences as a basis for program design (Kennedy, 2016). A differentiated approach includes the need for professional development activities that vary according to the affordances and constraints of context, including available time, cost, the scope of the change, and the available support. This personalised approach also includes taking into consideration the needs and interests of teachers and recognition of their different strengths (Tomlinson, 2005). Active

school leadership that involves teachers, students, and the broader community as partners in the change process promotes teacher ownership of the professional learning process, thereby increasing its effectiveness (AITSL, n.d.).

Guskey (2000) proposed an evaluative framework that included participants' reactions, participants learning, organisation support and change, participants' use of new knowledge and skills, and student learning outcomes. Driving questions formed the basis of this framework at each level. He proposed that in planning professional development to improve student learning, the framework should be used in reverse, promoting a backward planning approach. This approach draws focus away from what to do (workshops, seminars) or how to do it (study groups, peer coaching) to what professional learning developers want to achieve in terms of learning and learners (Guskey, 2002).

2.4.7 Strategies for professional learning

Considering the research base guiding effective professional development design and effectiveness, the specific strategies to enable particular goals become relevant for schools planning and implementing change. Loucks-Horsley, Love, Stiles, Mundry, and Hewson (2010) outlined effective professional learning strategies based on their analysis of a range of empirical and evaluative studies of professional development and mathematics education. The 18 strategies were organised into six clusters: aligning and implementing curriculum, collaborative structures, examining teaching and learning, immersion experiences, practising teaching, and vehicles and mechanisms. The strategies in each cluster share assumptions about teaching, learning, and professional development. The use of one or more of these 18 strategies depends on the intended outcomes of the professional development, as well as where participants are in the change process (see summary in Table 2.1) This outline of professional development purposes and procedures is analogous to Shulman's (1986) model of pedagogical reasoning and action (comprehension, transformation, instruction, evaluation, reflection, new comprehensions). Like Shulman's model, it acknowledges the flexible nature of knowledge acquisition and the importance of reflection in this progressive process. Such processes guide teachers to construct knowledge in the same way as effective learning experiences for students (Loucks-Horsley et al., 2010).

Table 2.1

Purposes and Strategies for Professional Development Summary

Coaching Mentoring
Curriculum implementation
Demonstration lessons
Coaching Mentoring
Curriculum implementation
Demonstration lessons

Purposes for Professional Development

Details of Purpose Possible Strategies

Developing awareness Usually at beginning of change process Introduction of new approach or content Elicit thoughtful questioning about new information

Practising teaching Engaging with the new approach, practice, or pr
during instruction in clas

Examining student work Lesson study Coaching Mentoring Demonstration lessons

Professional networks Demonstration lessons Study groups

Reflect deeply on teaching and learning

Reflection on classroom experiences Assessing and reflecting on impact of changes on students Generating ideas for improvement

Building knowledge Develop content and pedagogical content knowledge

Case discussions Immersion experiences Workshops Technology for professional development Partnerships with mathematicians

Action research Study groups Lesson study Case discussion Examining student work
Action research Study groups Lesson study Case discussion Examining student work

Translate new knowledge into practice

raw on knowledge base to
an instruction and improve
eaching

Note. Adapted from Loucks-Horsley et al., 2010, pp. 114–115.

Immersion experiences and implementation of curriculum are suggested as possible strategies for developing PCK, then drawing on this new knowledge base to plan and improve instruction. As the current research occurs coincident with the introduction of a new national mathematics curriculum, the role of curriculum in relation to teacher practice is meaningful.

2.4.8 Summary of literature on teacher growth and change

The research cited acknowledges the need for professional learning opportunities that are immersed in teachers' everyday practice, that view teachers' learning as ongoing and connected

to their students' outcomes. Research that attempts to model the process of teacher change reflects current thought about the multiple and cyclic pathways that may occur, reflecting also the individual nature of teacher change. The importance of teacher reflection has been noted, although difficulties in promoting this beneficial process have been documented. Contributing factors like time, school context, teacher beliefs, and the availability of external expertise have also been raised as possible affordances and constraints in the change process. Continued interest in which strategies best promote teacher change and the features of effective professional learning programs that facilitate this provides stimulus for further investigation like that proposed in this study.

2.5 CURRICULUM AND ITS IMPLEMENTATION

The study of curriculum and curriculum implementation remains central in research attempting to improve learning opportunities for students (Reys, Reys, & Rubenstein, 2010). Its role in the professional growth of teachers has also been recognised over time (Fullan & Pomfret, 1977; Remillard, 2005; Stein & Kim, 2009). Reforms of mathematics curriculum, particularly in the US since the 1990s, have promoted interest in the teacher professional learning opportunities needed to support teachers meet the expectations of such reform. Because *reform-oriented* experiences demand that teachers rethink what it means to teach, curriculum implementation and support materials offer a promising way to initiate and support significant professional growth in many areas related to mathematics, teaching, and learning (Charalambous & Hill, 2012). It has been suggested that failure of curricula as a means to promote changes in teaching approach can be attributed to the lack of consideration for the central role of the teacher in the curriculum implementation process (Remillard, 2005). Clarity is needed in relation to the teacher–curriculum relationship to guide schools seeking to use curriculum as a change agent.

The term curriculum is understood differently depending on the context and the purpose for its use. Hjalmarson (2008) referred to its Latin origin *currere*, meaning *to run*, connecting it to a course of study that “runs over time and moves forward” (p. 592). Suggested here is curriculum as a “system of interactive components at different levels of the educational system” (p. 593), including districts, individual schools, teachers, and teaching materials. Curriculum is described by Clements (2007) as a “written instructional blueprint and set of materials for guiding students’ acquisition of certain cultural values, concepts, procedures, intellectual dispositions, and ways of reasoning” (p. 36). This definition points to the social constructs surrounding tangible objects and educational participants, including the interpretations and

expectations they carry with them. The sociocultural significance of curriculum was also referred to by Brown (2002) in his description of curriculum as an “artifact” or tool made by communities to “document and convey intended practices” (p. 18). In Australia, and the state of NSW in particular, the curriculum is often used interchangeably with the term syllabus, that is, the knowledge, skills, and understandings outlined in state or nationally sanctioned policy documents. This aspect of curriculum is often referred to in the research literature as *base* curriculum (Stein & Kim, 2009).

Fullan and Pomfret (1977) argued that curriculum implementation is a phenomenon of its own, not simply an extension of planning and adoption processes. They suggested it is necessary to examine implementation in its own right to analyse why some educational changes were adopted and established and others not. This examination includes separation of the determinants of implementation from the implementation itself to assist focused analysis and conceptualisation of implementation and the factors affecting it. Determinants include the in-servicing provided and the extent to which certain factors such as lack of time or support helped or hindered the process. Implementation includes records of teachers’ knowledge, acceptance, and agreement of the underlying philosophy of the introduced curriculum, teachers' perception of their capacity to teach the curriculum effectively, and the specific practices observed in class when the curriculum was implemented.

2.5.1 Transformations of the curriculum

Transformations of the curriculum are described by Stein et al. (2007) in three temporal phases: the written, intended, and enacted. This transformation is also referred to as the ideal, available, adopted, implemented, achieved, and tested (Clements, 2007). MacNab (2000) used the terms intended, implemented, and experienced. Each of these characterisations attempts to describe the passage curriculum makes from the printed page (syllabus and/or support materials) to the teacher's plans for instruction, then to the actual task implementation in the classroom. Studying this passage requires an understanding of how teachers construct the enacted curriculum and the meditating processes apparent in this process (Remillard, 2005).

Different stages of these transformations are more affected by particular stakeholders. The written or intended curriculum is designed to accomplish the goals of the curriculum; it is primarily under the control of curriculum designers who may or may not refer to the mandatory state and national standards. The enacted or implemented curriculum is most influenced by the teacher's interpretations and modifications (Hjalmarson, 2008). It is acknowledged, however, that transformations occur within and across phases through the complex interactive and

interpretative processes applied by teachers. The reasons why teachers transform curriculum are multifaceted, including teacher characteristics (beliefs and knowledge, orientations, professional identity), teaching context (time, local cultures, teacher support, student responses), and the curriculum itself (traditional vs innovative, sequencing, level of structure) (Stein et al., 2007). Transformations of the curriculum suggest a dynamic teacher-and-text relationship in which "teachers not only adapt and change curricula but also are changed by the curricula they use" (Philipp, 2007, p. 287). Failure to consider such interaction and a teacher's need to learn in order to use new materials has contributed to the uneven role curriculum materials have played in teaching reform (Ball & Cohen, 1996).

Curriculum is designed to assist in the transmission of desired educative goals. Curriculum documents serve as the artefact with which teachers interact in delivering these goals. In the passage from design to implementation, teachers form relationships with the curriculum based on their own knowledge and experience, giving it meaning through their actions and the representations they value. This research attempted to describe the implementation process of an innovative curriculum unit with specific details about how teachers' knowledge and beliefs change in this process.

2.5.2 Curriculum support materials and teacher professional growth

Curriculum support materials have been a recent focus for researchers attempting to theorise the role they might play in teacher learning (Davis, Palincsar, Smith, Arias, & Kademian, 2017; Stein & Kim, 2009). There is agreement that for professional growth to occur, curriculum and support materials should be of high quality in both content and instructional strategies (Loucks-Horsley et al., 2010; Superfine, Marshall, & Kelso, 2015). The possibility of professional growth is increased if support materials are clear about underlying goals (conceptual understanding) of set tasks and provide ways to anticipate what learners may think or do while engaging with them (Stein & Kim, 2009).

Instructional materials hold a central position in influencing individual teachers' work. Their reach in the school system further supports this position (Ball & Cohen, 1996), being a traditional and routine part of teaching mathematics (Remillard, 2005). Curriculum support materials are instructional guides created for teachers (not student textbooks) to address the content and pedagogical approach of curriculum designers. Research about teachers' use of curriculum support materials in mathematics emerged in earnest in the US in the mid to late

1990s in response to standards-based curriculum that emphasised mathematical thinking, reasoning, conceptual understanding, and problem-solving in realistic contexts. Such

characteristics prompted researchers to measure the effectiveness of such materials as they were seen as a medium for introducing new ideas and pedagogical approaches to large numbers of teachers and therefore their students (Stein & Kim, 2009). It was felt an understanding of what happened in classrooms at schools implementing reform (problem-based) curricula would assist other school districts to build on this experience (Orrill & Anthony, 2003).

Interaction with curriculum and support materials can be thought of as an immersion experience (Loucks-Horsley et al., 2010) that has the potential to build mathematical knowledge for teaching while in the *act of* teaching. Curriculum support materials that are designed to be educative address both teacher and student learning. They promote opportunities to apply new knowledge of content, teaching, and learning in “real time” during classroom instruction, as well as through “planning, lesson modification, assessment, collaboration with colleagues and communication with parents” (David & Krajcik, 2005, p. 3). Such materials have been seen as a potential medium to support teacher learning on a large scale (Ball & Cohen, 1996).

Variations in the support materials provided to teachers can accentuate differences in implementation. Well-written and supportive classroom materials may better position teachers to implement curriculum as intended by the authors, thereby increasing the potential to simultaneously position teachers as learners (Superfine et al., 2015). Philipp (2007) pointed to the positive effect of curriculum (and support materials) that speaks *to* the teacher rather than *through* them. These materials guide teachers about the underlying ideas of the tasks rather than just guiding their actions (Remillard, 2000). Superfine et al. (2015) suggested quality implementation of a curricular program requires instructional support with information beyond the main part of the written text for teachers (often in the margins of the teacher’s page) including questions to ask students, examples of student solution strategies, suggestions for conducting discussion of student work, answers to the tasks set, and information about the concepts underpinning tasks.

In their research about the role of educative materials in supporting reform-based practices in (high school) science education, Schneider and Krajcik (2002) provided teachers with curriculum materials designed to promote teacher learning. The support materials of these inquiry-based units included the content, pedagogy, and PCK teachers needed for the unit as a whole, as well as within and across lessons. The unit included short scenarios to illustrate the language and possible ways of students working within a lesson to support teachers to model intended practice. Concept mapping was also included as a lesson activity to promote conceptual connections for both teachers and students.

The *voice* and *look* of curriculum support materials are less noticed structural features that may also influence how a teacher interacts with curriculum materials (Remillard, 2005). The voice of curriculum materials involves how the authors communicate with the teachers and students (Herbel-Eisenmann, 2009; Remillard, 2002). This voice is primarily achieved through the presence of authoritative structures of grammatical choices like imperatives, pronouns, and modal verbs. Herbel-Eisenmann (2009) suggested that imperatives invite students and teachers to be active participants in the curriculum tasks (use, make, draw) or to explain their thinking. First-person (I, we), second-person (you), and third-person (he, she, it, they) pronouns assist in the construction of roles and relationships between the reader and writer of the text. The use of a second-person pronoun *you* and a verb like *you will find* suggests the authors are either controlling the common knowledge by telling the reader about themselves or defining and drawing attention to this knowledge. Second-person pronouns are also used with objects, disguising the authority of the authors through the promotion of inanimate objects that instruct; that is, *the graph shows you*. Adding to the voice and look of curriculum materials are visual representations (Remillard, 2000). These include visual representations that may contribute to the underlying mathematics and associated tasks or simply be used to add to its attraction (e.g., use of colour). Teachers and students bring meaning to these objective structures, interpreting them within the context of the instructive materials and their own experiences. Analysis of such structural features and the interactions teachers have with them are important considerations when analysing the impact such materials may have on teachers' growth and change.

Beyond simply adding new ideas to a teacher's toolkit, curriculum support materials can provide opportunities for teachers to draw on their new knowledge base to plan instruction and improve their teaching (Loucks-Horsley et al., 2010), promoting future autonomy in decision-making. They can assist teachers to anticipate and interpret student responses to tasks (Ball & Cohen, 1996), describe how and why students have particular ideas about content and how to address them (Collopy, 2003), and support knowledge of appropriate instructional representations (Schneider & Krajcik, 2002). Part of this application across contexts is helping teachers to "learn how to listen and interpret what students say" (Ball & Cohen, 1996, p. 7), including the ability to anticipate what students may do in response to a range of instructional activities. Such materials invoke a case or story situation with "rich descriptions of events that illustrate theory" (Schneider & Krajcik, 2002, p. 224), giving teachers access to the prototypes, precedents, and/ or parables suggested by Shulman (1986) as valuable in guiding the work of teachers.