Children's Mathematical Knowledge Prior to Starting School

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The introduction of the *Early Years Learning Framework* and the *Australian Curriculum – Mathematics* in Australian preschools and primary schools has caused early childhood educators to reconsider what may be appropriate levels of mathematics knowledge to expect from children as they start school. This paper reports on initial data from an extensive evaluation of an early mathematics intervention and asks whether the expectations of young children recorded in the *Australian Curriculum – Mathematics* are realistic.

Research over the past ten years has established that many children starting school are more mathematically capable than many mathematics curricula and text books assume (Bobis, 2002; Clarke, Clarke, & Cheeseman, 2006; Ginsburg & Seo, 2000; Hunting et al., 2012). This finding suggests that many children may be inadequately challenged by the mathematics tasks and instruction they experience in their first year of school, and that this may have a negative impact on their opportunity to thrive mathematically (Perry & Dockett, 2008). The introduction across Australia of the new *Australian Curriculum – Mathematics* (Australian Curriculum, Assessment and Reporting Authority (ACARA), 2013) and its prior-to-school equivalent, the *Early Years Learning Framework* (Department of Education, Employment and Workplace Relations (DEEWR), 2009) provides an unprecedented opportunity to ensure that the Foundation Standard designated in the curriculum for the first year of school is appropriate for the level of mathematics children starting school bring with them.

This paper uses the opportunity provided by the *Let's Count* mathematics program to provide insight into this issue. In particular, it examines the mathematics knowledge of 125 children in December, 2012, prior to their beginning school in 2013, and compares this knowledge to that designated by the *Australian Curriculum – Mathematics* Foundation Standard. The children are part of the *Let's Count* Longitudinal Evaluation being conducted by the authors.

Australian Curriculum – Mathematics Foundation Achievement Standard

The *Australian Curriculum - Mathematics* (ACARA, 2013) aims to provide a national framework for mathematics learning and teaching in Australia. The achievement standard for children at the end of their first year at school is:

> Students make connections between number names, numerals and quantities up to 10. They compare objects using mass, length and capacity. Students connect events and the days of the week. They explain the order and duration of events. They use appropriate language to describe location.
>
> Students count to and from 20 and order small collections. They group objects based on common characteristics and sort shapes and objects. Students answer simple questions to collect information.

This achievement is supported by the Foundation Year Proficiency Standards that focus on understanding, fluency, problem solving and reasoning:

> Understanding includes connecting names, numerals and quantities.

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Fluency includes readily counting numbers in sequences, continuing patterns, and comparing the lengths of objects.

Problem Solving includes using materials to model authentic problems, sorting objects, using familiar counting sequences to solve unfamiliar problems, and discussing the reasonableness of the answer.

Reasoning includes explaining comparisons of quantities, creating patterns, and explaining processes for indirect comparison of length.

It is against these Foundation Standards and Proficiencies that we shall compare the mathematical knowledge of the 125 children in the *Let's Count* Longitudinal Evaluation.

The *Let's Count* Longitudinal Evaluation

Let's Count (Perry & Gervasoni, 2012) is a new early mathematics program designed by The Smith Family and the authors to assist parents and family members to help their children aged 3-5 years to play, investigate and learn powerful mathematical ideas in ways that develop positive dispositions to learning and mathematical knowledge and skills. *Let's Count* is supported by the Origin Foundation and developed in partnership with Blackrock Investment Management, and was piloted in 2011 in five sites designated as experiencing social and economic disadvantage, in five states across Australia. In 2012/2013, The Smith Family has delivered a revised *Let's Count* program in six additional sites. The program involves two professional learning modules for educators. The themes of the modules are:

Module 1: Noticing and exploring everyday opportunities for mathematics
Module 2: Celebrating mathematics.

One aspect of evaluating the effectiveness of the *Let's Count* program is to measure children's mathematical growth across their preschool year and also to compare their knowledge prior to beginning school with a comparison group of children whose families had not participated in *Let's Count*. The 125 children in the comparison group were assessed in December, 2012 us ing the *Early Numeracy Interview* (Department of Education Employment and Training, 2001).

Assessing children's knowledge of school mathematics

The *Early Numeracy Interview* (Department of Education Employment and Training, 2001) was chosen to assess the children's mathematical knowledge because it is task-based, interactive, based on extensive research, designed for young children, and enables progress to be measured in nine domains. This interview was developed as part of the *Early Numeracy Research Project* (ENRP) (Clarke et al., 2002) and refined during the *Bridging the Numeracy Gap* project (Gervasoni et al., 2011). An additional section of the interview focuses on early mathematics concepts for children beginning school.

The principles underlying the construction of the tasks and the associated growth points in the *Early Numeracy Interview* were to:

- describe the development of mathematical knowledge and understanding in the first three years of school in a form and language that was useful for teachers;
- reflect the findings of relevant international and local research in mathematics (e.g., Fuson, 1992; Gould, 2000; Mulligan, 1998; Steffe, von Glasersfeld, Richards, & Cobb, 1983; Wright, Martland, & Stafford, 2000);
- reflect, where possible, the structure of mathematics;
- allow the mathematical knowledge of individuals and groups to be described; and

- enable a consideration of students who may be mathematically vulnerable.

The interview focuses on early mathematics concepts and another eight mathematical domains. There are four whole number domains (Counting, Place Value, Addition and Subtraction, and Multiplication and Division); two measurement domains (Time and Length); and two geometry domains (Properties of Shape and Visualisation). The assessment tasks in the interview take between 30-45 minutes for each student and were administered by independent, trained assessors who followed a detailed script. Each child completed about 50 tasks in total, and given success with one task, the assessor continued with the next tasks in a domain for as long as a child was successful, according to the script. The processes for validating the growth points, the interview items and the comparative achievement of students are described in full in Clarke et al. (2002).

A critical role for the assessor throughout the interview was to listen and observe the children, noting their responses, strategies and explanations while completing each task. These responses were recorded on a detailed record sheet and then independently coded to determine whether or not a response was correct; identify the strategy used to complete a task, and the growth point reached by a child overall in each domain. This information was entered into an SPSS database for analysis. Of particular interest for this study were the children's responses to tasks in the early mathematics concepts section and the initial tasks in each of the other eight domains.

Participants in the 2012 Comparison Group

The 125 children assessed in December, 2012 were all eligible to begin school in 2013 and attended four-year-old pre-school programs in 10 centres in two large regional cities in Australia. In each of the 10 centres, a maximum of 15 children, all with the appropriate consents, were assessed. By chance, more boys (56%) were assessed than girls (44%).

The Mathematics Knowledge of the 2012 *Let's Count* Comparison Group

Pre-school children participating in the *Let's Count* Longitudinal Evaluation were presented with all the tasks in the Early Mathematics Concepts or 'Detour' Section of the Early Numeracy Interview and also with tasks from the Whole Number, Measurement and Geometry Domains for as far as each child was successful. Analysis of the 125 children's responses in December 2012 is presented in the tables below. The task analyses were grouped according to the associated *Australian Curriculum - Mathematics* Foundation Year Standard or Proficiency. Each table shows the percentage of children who were successful with each task in December 2012, comparison percentages for 1438 children in the February/March 2001 ENRP First Year at School cohort (Clarke et al., 2006), and the associated *Australian Curriculum – Mathematics* Foundation Year Standard or Proficiency. Results for some tasks were not available for the ENRP group as indicated with 'na' in the Tables. Table 1 focuses on children's success with tasks involving small sets of objects, usually small plastic teddies. The tasks were all associated with the Foundation Standard: students make connections between number names, numerals and quantities up to 10.

Table 1
Percentage Success on Tasks with Small Sets

Tasks	Dec 2012 (*n*=125)	ENRP 2001 (*n*=1438)	Australian Curriculum Foundation Standard
Tasks with Small Sets			
Count a collection of 4 teddies	95	93	
Identify one of two groups as "more"	90	84	
Make a set of 5 teddies when asked	77	85	
Conserve 5 when rearranged by child	79	58	
Combine 5+3 blue teddies and total	75	na	
Make collection of 7 (when shown number 7)	63	na	Students make connections between number names, numerals and quantities up to 10.
Knows one less than 7 w hen 1 t eddy removed	61	na	
Knows one less than 7 without recounting	25	na	
Part Part Whole Tasks			
Show 6 fingers (usually 5 & 1)	79	78	
6 fingers 2nd way	27	20	
6 fingers 3rd way	10	8	
One to one Correspondence Task			
Know 5 straws needed when asked to put 1 straw in each of 5 cups	88	92	

The results for the *Let's Count* pre-schoolers suggest that about three-quarters of these children demonstrate the Standard before they even begin school. The ENRP Comparison Group results are similar. This finding is reinforced by the results in Table 2.

Table 2
Percentage Success in Subitising Tasks and Matching Numerals to Dots

Tasks	Dec 2012 (*n*=125)	ENRP 2001 (*n*=1438)	Australian Curriculum Foundation Standard
Subitising Tasks			
Recognise 0 without counting	81	82	
Recognise 2 without counting	94	95	
Recognise 3 without counting	83	84	
Recognise random 3 without counting	86	na	
Recognise 4 without counting	70	71	
Recognise random 4 without counting	50	na	
Recognise 5 without counting	44	43	Students make connections between number names, numerals and quantities up to 10.
Recognise 9 without counting	16	9	
Matching Numerals to Dots Tasks			
Match numeral to 0 dots	73	63	
Match numeral to 2 dots	90	86	
Match numeral to 3 dots	73	79	
Match numeral to 3 random dots	82	na	
Match numeral to 4 dots	73	77	
Match numeral to 4 random dots	69	na	
Match numeral to 5 dots	65	67	
Match numeral to 9 dots	38	41	

Table 2 shows the percentage of children able to recognise the number of dots on a card without counting them, and also their ability to match a numeral to the number of dots. These results highlight that the majority of students can recognise quantities up to about four items without counting and about one-sixth of pre-schoolers can subitise nine dots. The majority of students can also match numerals to the number of dots, although nine was much harder to match than the other numbers. This ability to recognise quantities without counting is an important ability for teachers to build upon when planning instruction, and is important for exploring pattern and structure.

The importance of pattern and structure in young children's mathematical learning is gaining increased attention. The Foundation proficiencies of fluency and reasoning focus in continuing and creating patterns. The data presented in Table 3 suggest that about three-quarters of children can match patterns when they begin school, and one third of children can continue and explain a pattern.

Table 3
Percentage Success in Pattern Tasks

Tasks	Dec 2012 (n=125)	ENRP 2001 (n=1438)	Australian Curriculum Foundation Standard
Pattern Tasks			
Name colours in pattern	98	94	Fluency Proficiency includes:
Match pattern	72	76	continuing patterns.
Continue pattern	34	31	Reasoning Proficiency includes:
Explain pattern	34	31	creating patterns.

While continuing, creating and describing patterns is likely to be a profitable aspect of instruction for most children in their Foundation year at school, many children need further challenge right from the start. The Foundation standard also focuses on students counting to and from 20 and ordering small collections. Several tasks focused on sequence counting, counting a larger collection of at least 20 items and ordering numerals. The percentage of students able to complete these tasks is presented in Table 4.

Table 4
Percentage Success with Counting and Ordering Numerals

Tasks	Dec 2012 (n=125)	ENRP 2001 (n=1438)	Australian Curriculum Foundation Standard
Counting Tasks			
Rote count to 10	87	na	
Rote count to 20	29	na	
Count a collection of at least 20 & when one item is removed knows total without recounting	8	na	Students count to and from 20 and order small collections.
Ordering Numbers Tasks			
Order numeral cards 1-9	48	46	
Order numeral cards 0-9	32	38	
Orders 3 one digit numbers	47	na	
Orders 3 two digit numbers	28	na	

The data suggest that the majority of pre-schoolers can count to 10 and at least one-quarter can forward count to 20, but not back from 20. However, few students could count

20 teddies successfully and also identify how many teddies remained when one teddy was removed. This focus on the cardinal value of 20 is a profitable area for instruction in the first year at school, but is not highlighted in the Foundation Standard.

Several tasks in the interview focused on measuring length and time. Table 5 highlights that many children beginning school are able to compare and order lengths, in line with the Foundation Standard, and are also aware of the purpose of a clock. Seventeen percent of children could read 2 o'clock and knew some days of the week and months.

Table 5
Percentage Success with Length and Time Measurement Tasks

Tasks	Dec 2012 (*n*=125)	ENRP 2001 (*n*=1438)	Australian Curriculum Foundation Standard
Length Measurement Tasks			
Ordering 3 candles smallest to largest	73	61	
Ordering 4 candles smallest to largest	54	50	Students compare objects using mass, length and capacity.
Accurately compares two lengths– string and stick	65	na	
Measures length using informal units	8	na	
Time Measurement Tasks			Students connect events and the days of the week
Aware of the purpose of a clock	83	na	
Knows some days/months & 2 o'clock	17	na	

Spatial reasoning is a key aspect of learning mathematics. The data presented in Table 6 shows children's success with tasks involving describing and interpreting locations, recognising the properties of shapes and using mental imagery to manipulate shapes.

Table 6
Percentage Success on Spatial Tasks

Tasks	Dec 2012 (*n*=125)	ENRP 2001 (*n*=1438)	Australian Curriculum Foundation Standard
Language of Location Tasks			
Beside	94	88	Students use appropriate language to describe location.
Behind	87	87	
In front of	91	83	
Properties of Shapes Tasks			
Knows square	85	na	
Knows circle	92	na	
Knows rectangle	74	na	Students group objects based on common characteristics and sort shapes and objects
Knows some triangles	83	na	
Knows all triangles	63	na	
Visualisation Tasks			
Identifies a reoriented rectangle in room	89	na	
Identifies and traces possible shapes when a shape is partially hidden.	16	na	

The data suggest that the *Let's Count* pre-schoolers were proficient in these aspects of mathematics and almost all children meet the Foundation Standard prior to beginning school. The most difficult task involved dynamic imagery, and 16 percent were successful.

The interview also includes a range of tasks involving calculations, although few students progressed far in these domains. Results for four calculation tasks are presented in Table 7. All tasks were presented orally and involved the use of materials.

Table 7
Percentage Success on Calculation Tasks Involving Materials (Teddies)

Tasks	Dec 2012 (*n*=125)	Australian Curriculum Foundation Standard
Calculation Tasks		Problem Solving Proficiency: using materials to model authentic problems, sorting objects, using familiar counting sequences to solve unfamiliar problems, and discussing the reasonableness of the answer
Adds 5+3 when screen over 5 removed	49	
Adds 9+4 when screen over 9 removed	25	
Calculates total for 2 teddies in 4 cars	48	
Divides 12 teddies between 4 mats	31	

Most children who were successful with the first three tasks worked out the answers by counting all the items one by one. A small number of students used the count on strategy. Most children solved the division task through grouping rather than sharing by ones. The results and the children's strategies indicate that a large group of children are able to meet the Foundation *problem solving proficiency* before beginning school.

Conclusion

It is well established that the play, exploration and engagement of young children in everyday activities involves much informal mathematical activity (Ginsburg & Seo, 2000; Hunting et al., 2012). Nevertheless, the data presented in this paper highlights the broad range of formal mathematics knowledge that many children construct prior to beginning school. While the results presented here highlight the diversity of this group of children, it is also apparent that children's everyday home and pre-school experiences prepare a large proportion of them well for the transition to learning mathematics at school.

Comparison of children's mathematics knowledge with the *Australian Curriculum – Mathematics* Foundation Year Standard and Proficiencies suggest that large numbers of children in both the *Let's Count* pre-school group and the ENRP Beginning School group met the Standard prior to beginning school. An implication of this finding is that teachers need to extend the Foundation mathematics curriculum right from the first day of school to sufficiently challenge and engage many children in mathematics learning. While teachers are skilled in differentiating instruction for children, these findings highlight the importance of this role.

It is also apparent that many children in the *Let's Count* group had well developed measurement and spatial reasoning that exceeded the Foundation Standard. Overall, it appears that the new *Australian Curriculum – Mathematics* Foundation Standard is neither sufficiently challenging for children nor adequate for signalling to teachers the type of experiences and instruction that are important. Whilst acknowledging that the *Australian Curriculum – Mathematics* encourages teachers to adjust curriculum and instruction to match children's knowledge, it also needs to recognise the mathematical capabilities of children when they begin school. The data presented in this paper would suggest that there is still some fine-tuning to be done in setting the Foundation Standard.

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